Optimal Contracts and Supply-Driven Recessions

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In models with financial frictions, state-contingent contracts stabilize the business cycle relative to contracts with predetermined rates. We show that this finding depends on whether predetermined rates are set in real or nominal terms. State-contingent contracts can amplify supply-driven recessions compared to contracts set in nominal terms.

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1 Introduction

Since the Great Recession, financial frictions have become a ubiquitous ingredient in general equilibrium models used for policy analysis. In models à la Kiyotaki and Moore (1997), or Bernanke et al. (1999), the worsening of aggregate conditions deteriorates the collateral values and debt capacity of businesses, forcing firms to cut production and thus deepening the decline in aggregate activity. This mechanism amplifies the effect of the initial shock. Subsequent research has demonstrated that the amplification result hinges on the assumption that firms are excluded from hedging or insurance opportunities. The literature has shown that the amplification disappears when borrowers have access to contracts in which repayment rates are contingent on the aggregate state (Krishnamurthy, 2003; Nikolov, 2014; Cao and Nie, 2017; Carlstrom et al., 2016; Dmitriev and Hoddenbagh, 2017; Di Tella, 2017). Contingent rates decline in response to an adverse shock, providing financial relief to constrained borrowers and mitigating the negative effect on the economy. These findings suggest that moving from fixed-rate contracts towards optimal adjustable-rate contracts might stabilize business cycles.

While the literature has compared contingent contracts with predetermined real-rate contracts, most debt obligations in the real world are based on nominal rates. For example, in the US more than 90% of loans have nominal fixed rates (Badarinza et al., 2018). Therefore, the more practical question is whether optimal state-contingent contracts amplify shocks compared to contracts with predetermined nominal rates. We address this question by studying contracts with predetermined real or nominal rates as well as state-contingent rates in a standard model with collateral constraints and nominal rigidities à la Iacoviello (2005), which builds on Kiyotaki and Moore (1997). We find that while state-contingent rates stabilize the economy’s response to technology shocks relative to predetermined real rates, they amplify supply-driven recessions when compared to predetermined nominal rates. Our contribution suggests that for the US economy moving from nominal predetermined rates towards state-contingent rates may amplify business cycles.

While surprising, the intuition for the result is simple. The strength of the collateral amplification channel depends on how the initial shock affects the wealth of the borrowing entrepreneurs. Entrepreneurial financial wealth is the value of collateral (housing) net of the value of debt obligations. A negative technology shock decreases real collateral values. With contingent rates, real repayment rates fall to provide consumption insurance to borrowers, thereby stabilizing their financial wealth in response to the shock. When repayment rates are instead predetermined in nominal terms, a negative technology shock reduces collateral values but also creates surprise inflation that erodes the real value of debt obligations. If the inflationary pressure is sufficiently strong, borrowers’ financial position may improve on impact, leading to smaller recessions than in the case of state-contingent contracts. We show that this inflation-driven decrease in debt indeed outweighs the fall in real collateral values for a realistic calibration of the model.


2 CREDIT FRICTIONS IN GENERAL EQUILIBRIUM

We embed state-contingent loan contracts in a New Keynesian model with collateral constraints à la Iacoviello (2005). The model consists of patient households, impatient entrepreneurs, retailers, and a central bank. Households work for entrepreneurs, who produce intermediate goods using labor and housing. Retailers bundle together the intermediate goods into a final consumption good.

PATIENT HOUSEHOLDS

Patient households maximize their lifetime expected utility

\[
\mathbb{E}_t \sum_{t=0}^{\infty} \left\{ \beta^t \left( \ln c_t' + j \ln h_t' - (L_t')^\eta / \eta \right) \right\},
\]

where \( \beta \) is the household discount factor, \( c_t' \) is household consumption, \( h_t' \) is housing, \( L_t' \) denotes hours worked. The household budget constraint is:

\[
c_t' + q_t \Delta h_t' + \frac{B_t}{P_t} + b_t = b_{t-1} r_t + \frac{B_{t-1}}{P_t} R_{t-1} + w_t' L_t' + F_t,
\]

where \( q_t \) denotes the real price of housing, \( b_t \) is the real amount of lending to entrepreneurs, \( w_t' \) is the real wage and \( F_t \) are lump-sum profits received from retailers. We denote with \( r_t \) the real interest rate paid by loans to entrepreneurs between \( t-1 \) and \( t \). As we specify below in the entrepreneurial problem, this interest rate can be state-contingent or predetermined, nominal or real depending on the model considered. \( P_t \) is the general price level and \( B_t \) is the quantity of riskless nominal bonds traded only among patient households that pay a predetermined interest rate \( R_t \). These bonds are in zero net supply and exist to describe the effect of monetary policy in the limiting case of a cashless economy. Households maximize utility (1) subject to (2), choosing consumption, entrepreneurial loans, nominal bonds, hours worked, and housing, leading to the following first-order conditions:

\[
1 = \mathbb{E}_t \left\{ \Lambda_{t,t+1} r_{t+1} \right\},
\]

\[
1 = R_t \mathbb{E}_t \left\{ \frac{\Lambda_{t,t+1}}{\pi_{t+1}} \right\},
\]

\[
w_t' = (L_t')^\eta - 1 c_t',
\]

\[
\frac{q_t}{c_t'} = \frac{j}{H_t'} + \beta \mathbb{E}_t \left\{ \frac{q_{t+1}}{c_{t+1}'} \right\},
\]

where we have defined the household’s stochastic discount factor \( \Lambda_{t,t+1} = \beta^t \frac{c_t'}{c_{t+1}'} \) and \( \pi_t = \frac{P_t}{P_{t-1}} \).
Entrepreneurs produce intermediate goods using a Cobb-Douglas production function combining technology $A_t$, labor $L'_t$, and real estate $h_{t-1}$:

$$Y_t = A_t h_{t-1}^{v'} (L'_t)^{1-v}. \quad (7)$$

The entrepreneur’s expected lifetime utility is given by

$$\mathbb{E}_t \sum_{t=0}^{\infty} \gamma^t \ln c_t, \quad (8)$$

where $c_t$ is entrepreneurial consumption, and $\gamma$ is a discount factor. The budget constraint is:

$$\frac{Y_t}{X_t} + b_t = c_t + q_t \Delta h_t + r_t b_{t-1} + \omega'_t L'_t, \quad (9)$$

where $b_t$ is the amount borrowed and $X_t$ is the markup of final over intermediate goods. The borrower can default at the end of the period $t$ on its loan $b_t$, in which case lenders repossess a fraction $m$ of the housing assets $h_t$ and sell them in the next period for a price $q_{t+1}$. The resulting borrowing constraint is:

$$b_t \leq m h_t \mathbb{E}_t (q_{t+1} \Lambda_{t,t+1}). \quad (10)$$

The presence of the household’s stochastic discount factor $\Lambda_{t,t+1}$ highlights that lenders value the resources obtained from repossessing the collateral at time $t+1$ according to their marginal utility in that period. Borrowers offer lenders the amount they would like to borrow $b_t$ and the repayment schedule $r_{t+1}$ subject to the participation constraint (3), as lenders have an outside option of consuming instead of lending.

Entrepreneurs maximize expected utility (8) subject to (9), (10), and (3). The first order conditions with respect to debt, housing, labor are:

$$\frac{1}{c_t} = \gamma \mathbb{E}_t \left\{ \frac{r_{t+1}}{c_{t+1}} \right\} + \lambda_t, \quad (11)$$

$$\frac{q_t}{c_t} = \mathbb{E}_t \left\{ \frac{\gamma}{c_{t+1}} \left[ \frac{Y_{t+1}}{X_{t+1} h_t} + q_{t+1} \right] + m \lambda_t q_{t+1} \Lambda_{t,t+1} \right\}, \quad (12)$$

$$\omega'_t = (1 - v) \frac{Y_t}{X_t L'_t}. \quad (13)$$

For the predetermined real rates and nominal rates, the repayment rates are defined in (14a) and (14b), respectively. When state-contingent contracts are available, the first order condition with
The condition (14a) defines the real predetermined rate for loans from patient households to entrepreneurs. Equations (3), (4), and (14a) imply that to the first-order approximation Fisher relationship between nominal rates, real interest rates, and inflation \( \hat{r}_t = \hat{R}_t - E_t \hat{\pi}_{t+1} \) holds. Condition (14b) defines the nominal predetermined rate for loans from patient households to entrepreneurs. Optimal holdings of nominal bonds traded between entrepreneurs and households (3), as well as the optimality condition for nominal bonds traded among patient households (4) lead to no arbitrage relation \( \hat{R}_t = R_t \).

Equation (14c) is the optimality condition with respect to the repayment rate when state-contingent contracts are available. This condition established the relationship between the stochastic discount factor of the lender, that of the borrower, and \( \zeta_t \), the Lagrangian multiplier for the participation constraint (3). We now state our main theoretical result.

Proposition 1 Under state-contingent contracts there is a perfect comovement of marginal utility of consumption between households and entrepreneurs:

\[
\hat{U}_{c,t+1} - E_t \hat{U}_{c,t+1} = \hat{U}_{c,t+1} - E_t \hat{U}_{c,t+1},
\]

where \( U_{c,t+1} = \frac{1}{\gamma_{c,t+1}} \) and \( U_{c,t+1} = \frac{1}{\gamma_{c,t+1}} \) and hatted variables denote log-deviations from the steady state.

Proof Log-linearize (14c) to obtain \( \hat{\Lambda}_{t,t+1} \). The difference between \( \hat{\Lambda}_{t,t+1} \) and \( E_t \hat{\Lambda}_{t,t+1} \) gives (15) after using \( \Lambda_{t,t+1} = \beta \frac{\gamma^*}{\gamma_{c,t+1}} \).

Proposition 1 means that if collateral constraints for borrowers get tighter, the repayment rate should go down to prevent borrowers’ consumption to fall relative to lenders’. Consequently, borrowers receive consumption insurance from lower interest rates in recessions.

Retailers

Retailers purchase intermediate goods from entrepreneurs, costlessly differentiate them, and resell them to final good aggregators. Retailer can reset their prices every period with probability \( 1 - \theta \). The optimal reset price \( P_{t}^{*} \) is given by the following equation:

\[
\sum_{k=0}^{\theta} E_t \left\{ \Lambda_{t,k} \left[ (1 - \epsilon) \frac{P_{t+k}^{*}}{P_{t+k}} + \epsilon \frac{1}{X_{t+k}} \right] \left( \frac{P_{t+k}^{*}}{P_{t+k}} \right)^{-\epsilon} Y_{t+k} \right\} = 0.
\]

(16)
The aggregate price level is defined by

\[ P_t = \left( \theta P_{t-1}^{1-\varepsilon} + (1 - \theta)(P_{t-1}^*)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}. \]  

(17)

**MONETARY POLICY**

The central bank conducts monetary policy through a Taylor Rule:

\[ \log(R_t / R) = \rho_R \log(R_{t-1} / R) + (1 - \rho_R)(\rho_x \log(\pi_{t-1} / \pi) + \rho_Y \log(Y_{t-1} / Y)), \]  

(18)

where \( R \) is the steady-state nominal interest rate.

**MARKET CLEARING**

Finally, we have market clearing in the goods, housing and nominal bonds markets:

\[ c_t + c'_t = Y_t, \]  

(19)

\[ h_t + h'_t = 1, \]  

(20)

\[ B_t = 0. \]  

(21)

**SHOCKS**

Technology shocks follow a standard AR(1) process.

\[ \log(A_t / A) = \rho_A \log(A_{t-1} / A) + e_A^t, \]  

(22)

where \( e_A^t \) denotes an exogenous shock.

**EQUILIBRIUM**

The model has 18 endogenous variables and 18 equations. The endogenous variables are: \( c_t, c'_t, \lambda_t, L_t, Y_t, b_t, u_t, r_t, P_t, P^*_t, X_t, q_t, R_t, h_t, h'_t, B_t, A_t \). Depending on the type of the contract, there will be an extra variable \( \tilde{r}_t, \tilde{R}_t, \) or \( \zeta_t \). The equations defining these endogenous variables are (3), (4), (5), (6), (7), (9), (10), (11), (12), (13), (16), (17), (18), (19), (20), (21), (22). In addition, the repayment rate takes the form of (14a), (14b), or (14c), specific to each contract. The models are log-linearized around the common steady state.

3 **SIMULATIONS**

Our quarterly calibration largely follows Iacoviello (2005) and is reported in Table 1. We set a steady-state markup of 20% as standard in the New Keynesian literature. As our model does not feature physical capital, we follow Nikolov (2014) in matching the share of tangible assets in GDP equal to 0.36 by setting the value of \( \nu = 0.23 \). We set the persistence \( \rho_A \) for the technology shock at 0.9.

\[ \nu = \frac{\nu}{X} + \frac{X-1}{X}. \]  

\footnote{The share of tangible assets consists of rental income and monopolistic profits, which add up to \( \frac{\nu}{X} + \frac{X-1}{X} \).}
We have found the result to be robust to alternative values of these parameters as well.

Figure 1 shows the impulse responses to a one-percent negative technology shock. The shock lowers housing productivity, which results in a decrease in housing prices and output in all three models considered. The Figure confirms previous findings in the costly enforcement (Krishnamurthy, 2003; Nikolov, 2014; Cao and Nie, 2017) and costly state verification (Carlstrom et al., 2016; Dmitriev and Hoddenbagh, 2017) literature that state-contingent contracts stabilize business cycle fluctuations relative to an environment with predetermined real rate contracts. Our contribution is to show that state-contingent contracts amplify fluctuations in output relative to contracts with nominal predetermined rates.

The intuition for the results is quite straightforward and is based on the notion of financial wealth, that we define, following Cao and Nie (2017), as:

$$\omega_t = \frac{q_t h_{t-1} - rr_t b_{t-1}}{q_t H}.$$ 

The variable $\omega_t$ represents the relative financial wealth of entrepreneurs, which comprises their housing wealth net of the value of their debt obligations. The relative financial wealth of households is $1 - \omega_t$. The response of $\omega_t$ is displayed in the top-right panel of Figure 1.

Amplification or stabilization relative to state-contingent contracts depends on the response of the distribution of wealth. State-contingent contracts provide consumption insurance to entrepreneurs, redistributing the losses from lower housing productivity to lenders by means of lower repayment rates on loans. This offsetting effect is absent in the case of real rate contracts. In this case, repayment rates are entirely predetermined, so that falling housing prices necessarily erode entrepreneurial financial wealth, setting in motion a vicious cycle of falling entrepreneurial housing demand and collateral prices that ultimately amplify the effect of the initial shock on output.

\[^2\text{Financial wealth moves a little because our model features variable markups, and, hence, profits. Under flexible prices, profits would be always proportional to output and relative financial wealth would not respond at all to the shock. This is consistent with the findings in Cao and Nie (2017).}\]
By contrast, under nominal rate contracts, the negative technology shock weakens collateral values but also reduces the real value of debt by creating surprise inflation. Under the standard calibration that we have chosen the surprise inflation outweighs the fall in housing prices. As a result, entrepreneurial financial wealth increases, thus dampening the effect of the initial shocks. Thus, the rise in the relative financial wealth of entrepreneurs leads to smaller output fluctuations than under state-contingent contracts. Our findings thus suggest that state-contingent contracts may amplify business cycle fluctuations. This is particularly likely if adverse supply-side shocks have inflationary effects that outweigh the fall in the prices of productive assets such as housing.

4 Conclusion

We examine whether state-contingent contracts stabilize business cycles relative to debt instruments with predetermined rates using a standard model with collateral constraint and nominal rigidities. In contrast with the previous literature that has focused on contracts based on predetermined real rates, we focus on the empirically more relevant case of nominal contracts. We find that state-contingent contracts may amplify output fluctuations in comparison to contracts where interest rates are preset in nominal terms.
References


