Contests with sequential moves:
An experimental study

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Abstract
We study experimentally contests in which players make investment decisions sequentially, and information on prior investments is revealed between stages. Using a between-subject design, we consider all possible sequences in contests of three players and test two major comparative statics of the subgame-perfect Nash equilibrium: The positive effect of information disclosure on aggregate investment and earlier mover advantage. The former prediction is decidedly rejected, as we observe a reduction in aggregate investment when more information is sequentially disclosed. The evidence on earlier mover advantage is mixed but mostly does not support theory as well. Both predictions rely critically on forward-looking, sophisticated decision-making, which is not typical for our subjects.

Keywords: contest, sequential moves, experiment
JEL classification codes: C72, C99, D82, D91

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1 Introduction

In contests, agents spend resources attempting to secure a valuable prize. Examples include R&D competition, lobbying, political campaigns, competition for promotion or bonuses in organizations, and sports. Many contests are sequential, with some agents making investment decisions later than others. In this case, information about prior investments, if it is revealed, can play a crucial role.

In this paper, we study experimentally sequential contests in which players make investments in stages, and after each stage prior investments are revealed. Hinnosaar (2019) has recently proposed a novel approach to the subgame-perfect Nash equilibrium (SPNE) analysis of sequential contests that yields two major predictions. First, aggregate investment should increase with the amount of information revealed. That is, for a given total number of participants, introducing more sequential moves leads to an increase in aggregate investment. Second, there is a universal earlier mover advantage: In any sequential contest earlier movers invest more, win more often and earn higher payoffs than later movers. We test these predictions using a laboratory experiment.

In the experiment, we utilize a between-subject design to compare behavior in all possible sequential contests of three players. There are four treatments, (3), (1,2), (2,1) and (1,1,1), where the dimension of each vector \((n_1, \ldots, n_T)\) is the number of stages and components \(n_t\) give the number of players making investment decisions at stage \(t\). Treatment (3) is the standard simultaneous-move contest, (1,1,1) is fully sequential, and the other two treatments are in between. We use the lottery contest success function (CSF) of Tullock (1980) to determine the probability of each player winning given the final vector of investments. In the SPNE, aggregate investment increases in the number of stages and earlier-mover advantage is predicted in all sequential treatments.

The idea that earlier movers have a strategic advantage in markets goes back at least to Stackelberg who studied sequential duopoly with linear demand.\(^1\) However, due to a peculiar shape of best responses, there is no impact of sequential moves on equilibrium investment in two-stage Tullock contests of two players (Linster, 1993).\(^2\) Thus, three is the minimum number of players for which sequential moves produce any differences in equilibrium. Exploring multi-stage contests with more than two players is technically challenging, and until recently only results for several special cases were available (Glazer and Hassin, 2000; Kahana and Klunover, 2018; Dixit, 1987). The inverse best response approach of Hinnosaar (2019) allows for a systematic analysis of a wide class of arbitrary

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\(^1\) Von Stackelberg (2010) is the first English translation of the original book published in German in 1934.

\(^2\) The first-mover advantage arises in two-stage contests of more than two players (Dixit, 1987).
multi-stage games, including contests.

While the experimental literature on behavior in contests is vast (for a review see, e.g., Dechenaux, Kovenock and Sheremeta, 2015), the literature on sequential contests is rather scarce. Fonseca (2009) compared behavior in two-player sequential and simultaneous-move contests, focusing primarily on the effect of heterogeneity in players’ abilities. In his baseline treatments with symmetric players, Fonseca (2009) observed behavior consistent with the neutrality prediction of Linster (1993), i.e., no differences in investment between simultaneous and sequential contests. Nelson (2019) studied entry deterrence in two-player and three-player sequential contests. The goal of that study was to test the prediction of Linster (1993) and its breakdown when transitioning from two to three players. However, motivated by third-party entry in political competition with two stable incumbent parties, Nelson (2019) only considered one type of three-player sequential contests – those involving two first movers.\(^3\) In the present paper, we reuse data from two three-player treatments of Nelson (2019) – simultaneous and (2,1) – and add two other treatments – (1,2) and (1,1,1) – to consider all possible move sequences and directly test the predictions of Hinnosaar (2019).

The rest of the paper is structured as follows. Section 2 summarizes the theoretical model and main predictions, and Section 3 presents the experimental design. Results are reported in Section 4 and discussed in Section 5.

## 2 Theory and predictions

We consider sequential contests of \(n = 3\) symmetric, risk-neutral players \(i = 1, 2, 3\) choosing investments \(x_i \in \mathbb{R}_+\). Investments are made in stages, and at the beginning of each stage all prior investments are revealed. The winner of the contest is determined after all three players made their investment decisions. The probability of player \(i\) winning is given by the lottery contest success function (CSF) of Tullock (1980),

\[
p_i = \begin{cases} 
\frac{x_i}{\sum_{j=1}^{n} x_j}, & \text{if } \sum_{j=1}^{n} x_j > 0 \\
\frac{1}{n}, & \text{if } \sum_{j=1}^{n} x_j = 0 
\end{cases} \tag{1}
\]

The winner receives a prize \(V > 0\), and all players lose their investments.

A sequence of moves in a contest of \(n\) symmetric players is completely characterized by a \(T\)-dimensional vector \(\mathbf{n} = (n_1, \ldots, n_T)\), where \(T \geq 1\) is the number of stages, \(n_t \geq 1\) is the number of players making investment decisions at stage \(t\), and \(\sum_{t=1}^{T} n_t = n\). For

\(^3\)Sequence (2,1), using he taxonomy of treatments in this paper.
There are four possible sequences: (3) is a one-stage simultaneous-move contest, (1,2) and (2,1) are two-stage Stackelberg-like contests with one leader and two followers, and with two leaders and one follower, respectively; finally, (1,1,1) is a three-stage contest where one player makes a decision at each stage.

The solution concept is subgame-perfect Nash equilibrium (SPNE). Let \( X \) denote aggregate equilibrium investment in a sequential contest \( n \) with unit prize. Following the inverse best response method developed by Hinnosaar (2019), \( X \) can be calculated as the largest root of the equation \( f_0(X) = 0 \), where functions \( f_0, f_1, \ldots, f_T \) are obtained recursively using the relation \( f_{t-1}(X) = f_t(X) - n_t f'_t(X) X (1 - X) \), with terminal condition \( f_T(X) = X \). Individual equilibrium investment of players making decisions at stage \( t \) can then be found as \( x^*_t = \frac{1}{n_t} [f_t(X) - f_{t-1}(X)] \). In the experiment, we use \( n = 3 \) and prize \( V = 240 \). The resulting individual and aggregate equilibrium investment is shown in Table 1. We follow the convention that player \( i \) (weakly) precedes player \( j \) for \( i < j \).

There are two main predictions: First, aggregate effort is increasing as more information is revealed. Second, in sequential treatments earlier movers have an advantage in terms of investment, winning probabilities and payoffs. We test these predictions using the experiment described next.

### 3 Experimental design

**Preliminaries** The experiment followed a between-subject design with four treatments corresponding to the four possible move sequences in three-player contests. A total of 333 subjects (64\% of them female) were recruited using ORSEE (Greiner, 2015) from the population of \( \sim 3000 \) students at Florida State University who pre-registered for participation in social science experiments. There were 81, 90, 81 and 81 subjects in treatments (3), (1,2), (2,1) and (1,1,1), respectively. A total of 21 sessions were conducted in the XSFS Lab at FSU, with subjects making decisions at visually separated computer terminals. The experiment was implemented in zTree (Fischbacher, 2007). On average, sessions lasted \( \sim 60 \) minutes, and subjects earned $21.17 including a $7 participation fee.

**Procedures** Each session consisted of three parts. Instructions for each part were distributed on paper at the beginning of that part and read out loud by the experimenter (sample instructions are available in Appendix A). In Part 1, subjects’ risk attitudes were assessed using the method of Holt and Laury (2002). Subjects made 10 choices between two lotteries, \( A = (\$2.00, \$1.60; p, 1 - p) \) and \( B = (\$3.85, \$0.10; p, 1 - p) \), with \( p \) taking values 0.1, 0.2, \ldots, 1.0. One of the 10 choices was selected randomly and played out at the
Table 1: Equilibrium predictions (SPNE) and summary statistics. Standard errors clustered by matching group in parentheses.

<table>
<thead>
<tr>
<th>Treatments</th>
<th>(3)</th>
<th>(1,2)</th>
<th>(2,1)</th>
<th>(1,1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPNE</td>
<td>Observed SPNE</td>
<td>Observed</td>
<td>Observed</td>
<td>Observed</td>
</tr>
<tr>
<td>$x_1$</td>
<td>53.33</td>
<td>84.85</td>
<td>90</td>
<td>88.98</td>
</tr>
<tr>
<td></td>
<td>(6.29)</td>
<td>(8.89)</td>
<td>(7.24)</td>
<td>(6.35)</td>
</tr>
<tr>
<td>$x_2$</td>
<td>53.33</td>
<td>84.85</td>
<td>45</td>
<td>70.84</td>
</tr>
<tr>
<td></td>
<td>(6.29)</td>
<td>(3.91)</td>
<td>(7.24)</td>
<td>(7.72)</td>
</tr>
<tr>
<td>$x_3$</td>
<td>53.33</td>
<td>84.85</td>
<td>45</td>
<td>70.84</td>
</tr>
<tr>
<td></td>
<td>(6.29)</td>
<td>(3.91)</td>
<td>(7.39)</td>
<td>(6.12)</td>
</tr>
<tr>
<td>$X$</td>
<td>160</td>
<td>254.54</td>
<td>180</td>
<td>230.65</td>
</tr>
<tr>
<td></td>
<td>(18.88)</td>
<td>(13.96)</td>
<td>(13.50)</td>
<td>(15.32)</td>
</tr>
</tbody>
</table>

end of the session.

In Part 2 – the main part of the experiment – subjects were randomly divided into fixed matching groups of 9 and only interacted within those groups. Subjects went through 25 identical rounds of the contest game with moves sequenced, and information revealed across stages, according to the treatment. At the beginning of each round, subjects within each matching group were randomly matched into groups of three. Each subject was given an endowment of 240 points and could invest any integer number of points between 0 and 240 into the contest. After all subjects made their investment decisions, one winner within each group was randomly determined according to CSF (1). The winner received a prize of 240 points, and all subjects lost their investments. One round of the 25 was chosen at the end of the session to base subjects’ actual earnings on, at the exchange rate of 20 points = 1$.

In Part 3, we administered a short questionnaire. Subjects reported their gender, age, and self-assessed competitiveness measured on a Likert scale from 1 (“much less competitive than average”) to 5 (“much more competitive than average”). After Part 3, earnings from all parts were calculated and revealed, and subjects were paid privately by check.

4 Results

Table 1 shows average individual and aggregate investments by treatment, along with the equilibrium predictions. Individual investment $x_i$ is indexed by $i = 1, 2, 3$, where we follow the convention that player $i$ makes a decision no later than player $j$ for $i < j$. For example, in treatment (2,1), $x_1$ and $x_2$ are the investments of two players at the first
Figure 1: Aggregate investment, by treatment. The black lines show the SPNE predictions.

stage and \( x_3 \) is the investment of the third player at the second stage. In what follows, we employ both parametric and nonparametric tests, with clustering at the matching group level in the former and matching group as the unit of observation in the latter.

4.1 Aggregate investment

We start with the analysis of aggregate investment, \( X \). As seen from Table 1, the observed averages exceed equilibrium predictions in all treatments. The difference is statistically significant in all treatments except \((1,1,1)\) \((p = 0.001, 0.006, 0.019 \text{ and } 0.295 \text{ for } (3), (1,2), (2,1) \text{ and } (1,1,1), \text{ respectively; the Wald test})\). While excessive investment (often termed “overbidding”) is rather standard for contest experiments (see, e.g., a review by Sheremeta, 2013), we note that overbidding declines as more information is revealed.

This decline in aggregate overbidding is due to the fact that, as more information is revealed, the equilibrium aggregate investment (weakly) increases while the observed investment decreases, cf. Figure 1. The decline in investment from one treatment to the next is not large enough for statistical significance, but the overall decline from \((3)\) to \((2,1)\) and from \((3)\) to \((1,1,1)\) is significant, especially in later rounds. Two-sided \(p\)-values for pairwise comparisons using data from all rounds, last 10 rounds, and last 5 rounds are shown in Table 2.

**Result 1** Aggregate investment decreases as more information is revealed. The effect is statistically significant for comparisons between \((3)\) and \((1,1,1)\) and between \((3)\) and \((2,1)\),
**Table 2:** Two-sided $p$-values for comparisons of aggregate investment using the Wald test with errors clustered by matching group, and the Mann-Whitney U test with matching group as the unit of observation.

Result 1 is in striking contradiction to the first major theoretical prediction of Hinnosaar (2019) – that aggregate investment should increase with the amount of information. We look at individual investment for clues on why we observe this pattern.

### 4.2 Individual investment

Average individual investment by treatment and role is shown in Figure 2. First, we compare individual investment to equilibrium predictions. In the simultaneous treatment (3), there is significant overbidding of about 60% ($p = 0.001$, the Wald test), consistent with most other contest experiments. In contrast, in sequential treatments whether or not overbidding is present depends on the role. In (1,2) and (2,1), first movers’ investment is in line with the equilibrium ($p = 0.911$ and 0.289, respectively), while second movers overbid significantly ($p = 0.000$ and 0.014, respectively). In (1,1,1), first movers *underbid* ($p = 0.009$), second movers are in line with the equilibrium ($p = 0.930$), and third movers overbid ($p = 0.000$). While overbidding by second movers in (1,2) and (2,1) is about 60%, in line with the simultaneous move treatment, third movers in (1,1,1) overbid close to 100%.

Comparing investment by players in different roles within each of the sequential treatments, we find that first movers bid more than second movers in (1,2) ($p = 0.054$), but not in (2,1) ($p = 0.559$) or (1,1,1) ($p = 0.766$). Third movers in (1,1,1) bid more than both first and second movers ($p = 0.083$ and 0.082, respectively).

**Result 2** *Earlier movers’ advantage is observed in (1,2), but not in (2,1) or (1,1,1). In the latter treatment, the last mover has an advantage.*

Result 2 provides mixed evidence on the support for the second major prediction of (Hinnosaar, 2019) – that earlier movers should have an advantage in sequential contests. The behavior in (1,2) is in line with the prediction, while in (1,1,1) it is essentially the
5 Discussion

There is a general pattern of individual investment in sequential treatments (1,2), (2,1) and (1,1,1): Earlier movers are cautious and later movers are overly aggressive. Anticipation of their investment being revealed at later stages drives the behavior of the former, while the actual information on prior investments leads the latter.

We can distinguish between the strategic value and substantive value of information revealed between stages. In the SPNE, information does not have a substantive value; indeed, fully rational, backward-inducting players anticipate the decisions of later movers and best-respond to them. Later movers, in turn, best respond to the decisions of earlier movers. No uncertainty is resolved across stages, and information revelation does not update anyone’s beliefs (assuming unbounded rationality to begin with).

In contrast, the strategic value of information in the SPNE is high. Note that the simultaneous move equilibrium is still the Nash equilibrium (NE) of the sequential game; however, it is not subgame-perfect. The very fact that decisions are made in stages and information is going to be revealed makes the threat of playing the NE by later movers non-credible and produces the advantage for early movers. It also raises aggregate
investment, more so the more information is revealed.

However, as more information is revealed the game becomes longer and more rounds of backward induction are needed to recognize the SPNE. Think of boundedly rational players who cannot fully backward-induct and/or make mistakes in best response calculations. Such players view the behavior of other players as uncertain and would rather be later movers to resolve this uncertainty, at least partially. For such players, the strategic value of information is negative, but there is a positive substantive value.

It is well documented that human subjects are generally bad at backward induction, especially in longer games (e.g., McKelvey and Palfrey, 1992; Fey, McKelvey and Palfrey, 1996; Binmore et al., 2002), although hyper-rational subjects may be very good at it (e.g., Palacios-Huerta and Volij, 2009). Our results are in line with this observation. The contest game is arguably more complex than the centipede or alternating offer bargaining games these classic studies are based on.

Our results suggest that the SPNE predictions do not work because, essentially, our subjects are not sophisticated enough. The strong predictions of Hinnosaar (2019) regarding the effect of information revelation on aggregate investment and early mover advantage rely entirely on subgame perfection. Instead, our subjects use the information to resolve uncertainty in a strategically ambiguous environment. The effect is most pronounced in the longest game (1,1,1) where we observe later mover advantage. The cautious behavior of early movers also drives aggregate investment down.

This study was designed to only produce basic tests of the comparative statics of the SPNE, and cannot address the underlying causes too deeply. Our attempts to explain subjects’ behavior via individual characteristics, such as gender, risk-aversion, self-assessed competitiveness or field of study, did not produce any systematic results. Further investigation of sequential contests studying longer games, carefully controlling for strategic sophistication, or looking at various interventions helping subjects behave more strategically, can be of interest. That said, the fact that the key policy relevant prediction – the effect of interim disclosure on aggregate investment – is reversed already in the simplest nontrivial contest environment, is somewhat disheartening.

References


A Experimental instructions

General Information

Welcome to today’s experiment. Please refrain from making noise or communicating with the other participants during the experiment. If you have any questions please raise your hand and someone will come and answer your question privately.

For your participation in this experiment you will receive a show up payment of $7.00, and will have the opportunity to earn additional money during the experiment. Your payments will depend on the decisions made by you and the other participants. At the end of the experiment you will be paid privately. No other participant will be made aware of your payment.

There will be several stages to this experiment and you will be provided instructions prior to the beginning of each stage.

Part 1

For your first task today you will be asked to make a choice between a series of lotteries. You will be shown a list of ten pairs of lotteries as shown below:

Figure 3
Each option has two possible payoffs and an associated probability of getting that payoff. For example in row 2 Option A has a \( \frac{2}{10} \) or 20% chance of paying out $2.00 and a \( \frac{8}{10} \) or 80% chance of paying out $1.60. In the same row Option B has a \( \frac{2}{10} \) or 20% chance of paying out $3.85 and a \( \frac{8}{10} \) or 80% chance of paying out $0.10.

In row 7 Option A has a \( \frac{7}{10} \) or 70% chance of paying out $2.00 and a \( \frac{3}{10} \) or 30% chance of paying out $1.60. In the same row Option B has a \( \frac{7}{10} \) or 70% chance of paying out $3.85 and a \( \frac{3}{10} \) or 30% chance of paying out $0.10.

For each pair you will choose either Option A or Option B. After you have made your choices between the lotteries click submit to finalize your decision. Your choices will determine your payout for this part of the experiment via a random process.

Two random numbers between 1 and 10 have been pre-drawn and written on a piece of paper in an envelope that will be placed in the front of the room. At the end of the experiment the envelope will be opened and the numbers will be revealed. The first number determines which of the ten pairs are chosen. The second number determines the outcome. If the second random number is equal to or less than the numerator of the first probability then the first value of your choice will be paid out, if the random number is greater than the numerator then the second value will be paid out.

For example, if the first number is 3 and you chose Option A in the third row you would be in a lottery with a \( \frac{3}{10} \) or 30% chance of paying out $2.00 and a \( \frac{7}{10} \) or 70% chance of paying out $1.60. If the second number is a 1, 2, or 3, you would receive $2.00, if the second number is a 4, 5, 6, 7, 8, 9, or 10 you would receive $1.60.

Are there any questions before you make your decisions?

Part 2

All amounts in this portion of the experiment will be expressed in points. The exchange rate will be 100 points = $5.00 or 1 point = $0.05. This part of the experiment will have you make decisions over 25 rounds.

Endowment and Expenditure

At the beginning of this part of the experiment you will be randomly assigned one of three possible roles: Leader, Follower 1 or Follower 2. You will maintain the same role throughout this part of the experiment. In each round you will be randomly matched with two other participants, and given an endowment of 240 points to expend on a contest.

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4These instructions are for treatment (1,1,1). Instructions for other treatments are similar and available from the authors upon request.
Every group will have 1 player of each type. Leader will make his or her expenditure decision first, which will then be revealed to Follower 1, who then decides his or her expenditure. Follower 2 then sees the expenditures of Leader and Follower 1 and decides his or her expenditure. You can expend any integer number of points from 0 to 240. You will keep any points you choose not to expend. If you win the prize for the round you will receive 240 additional points.

**Probability of Winning**

After the expenditure decisions are made the sum of expenditures in your group will be calculated. Then the probability of you winning the prize in that round is given by:

\[
\frac{\text{Your Expenditure}}{\text{The Sum of Your Group's Expenditures}}
\]

For example, suppose you chose to expend 20 points and another member of your group chose to expend 30 points, and the third member of your group chose to expend 50 points. Then the probability you will win the prize is:

\[
\frac{20}{20 + 30 + 50} = \frac{20}{100} = \frac{1}{5} = 20\%
\]

For another example, suppose you chose to expend 100 points and another member of your group chose to expend 20 points, and the third member of your group chose to expend 40 points. Then the probability you will win the prize is:

\[
\frac{100}{100 + 20 + 40} = \frac{100}{160} = \frac{5}{8} = 62.5\%
\]

**Payoff in a Given Round**

After determining the probability that you win the computer will randomly assign you to a player number. The first player will be assigned to the interval from 0 to their probability of winning, player 2 will receive the interval from player 1’s probability of winning to player 1’s probability of winning plus player 2’s probability of winning, player 3 will be assigned the interval from player 1’s probability of winning plus player 2’s probability of winning to 100. The computer then draws a random number between 0 and 100 to determine which member of your group wins the prize. If the number drawn is in your interval, you will win the prize, otherwise another player in your group will win the prize. Using the first example from above, if the player numbers are assigned in the order of expenditures then player 1 has the interval from 0 to 20, player 2 has the interval from 20 to 50, and player 3 has the interval from 50 to 100. If the number drawn is less than
or equal to 20 player 1 wins, if it is larger than 20 but less than 50 player 2 wins, and if it is greater than 50 player 3 wins.

The individual payoff is then calculated as follows:

<table>
<thead>
<tr>
<th>If you win:</th>
<th>If you lose:</th>
</tr>
</thead>
<tbody>
<tr>
<td>240 (Endowment)</td>
<td>240 (Endowment)</td>
</tr>
<tr>
<td>240 (Prize)</td>
<td>-(Expenditure)</td>
</tr>
<tr>
<td>-(Expenditure)</td>
<td></td>
</tr>
<tr>
<td>480-Expenditure</td>
<td></td>
</tr>
</tbody>
</table>

Your earnings

You will compete in a series of 25 rounds, and will be paid your payoff for one of them, chosen at random. Prior to the experiment beginning a random number was drawn and placed in an envelope in front of the class. That number contains the round you will be paid for. At the end of the experiment the round number you were paid for and the payout will be displayed for you to confirm the output. Are there any questions at this time?

We will begin with a non paying practice round to familiarize yourself with the controls. For this round only you can input other players’ decisions and observe your probability of winning the prize.

Practice

Before you start making decisions in part 2 of the experiment, we will go over a practice round. These decisions do not affect your earnings and are designed to allow you to become familiar with the interface. In this practice part only, you will be able to enter decisions both for yourself and for other members of your group.

To begin with try entering a value of 50 for each players’ expenditure. You will see that the probability of winning is 1/3.

Using an example from the instructions you would enter 100 points for yourself, 20 points for one other member of your group and 40 points for the final group member. The probability that is displayed is 62.5%
\[
\frac{100}{100 + 20 + 40} = \frac{100}{160} = \frac{5}{8} = 62.5\%
\]

Take a few moments to input values and see how the probability of winning changes. When you are ready to continue press the continue button at the bottom of the screen.