Resurrecting the Weak Credibility Hypothesis in Models of Exchange-Rate-Based Stabilization

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ABSTRACT

We analyze how weak credibility affects the volatility of consumption spending in a model of exchange-rate-based stabilization that allows for both durable and nondurable goods. The inclusion of durables greatly improves the explanatory power of the weak credibility hypothesis. The hypothesis can account for the main qualitative properties of the boom-bust cycle provided the elasticity of durables expenditure with respect to Tobin’s q is greater than the intertemporal elasticity of substitution. Moreover, the quantitative effects are very large. In numerical simulations based on conservative assumptions about the expenditure share of durables (20%) and wealth effects (none), aggregate consumption increases 12-28% during the low-crawl phase and the real exchange rate appreciates 24-26%. In variants of the model that incorporate supply effects, the consumption boom is equally strong but appreciation of the real exchange rate rises to 30-40%.

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The announcement of an exchange-rate-based stabilization (ERBS) program is usually followed by a pronounced surge in consumption spending. This stylized fact is the key to understanding many of the other stylized facts associated with ERBS. It is a small step from a consumption boom to a large current account deficit, large capital inflows, and persistent, strong appreciation of the real exchange rate. If prices are sticky and firms in the nontradables sector produce to demand, the consumption boom also fuels a temporary output boom.

In a pair of seminal papers, Calvo and Vegh (1993, 1994a) focused on weak credibility as the underlying source of the consumption boom. The link between credibility and spending arises when holdings of real money balances affect the cost of consumption. Following Calvo and Vegh, suppose money demand is governed by a cash-in-advance constraint and that the country operates in a perfect world capital market. In this setup, a temporary (i.e., non-credible) reduction in the rate of crawl lowers the price of consumption today relative to the price of consumption in the future. Intertemporal substitution then leads to a consumption boom and a current account deficit financed by private capital inflows. The bill for high spending during the boom phase is paid in perpetuity in the post-ERBS period: when the policy collapses, consumption drops below its previous level and the country runs a trade surplus year in and year out to cover higher interest payments on the external debt.\footnote{While the weak credibility (WC) hypothesis exercises a strong intuitive appeal, its explanatory power is thought to be limited by the fact that the intertemporal elasticity of substitution is low in LDCs. Consider the solution for the peak increase in real consumption \((C_p)\) in the Calvo-Vegh model. This is\footnote{\[
\frac{C_p - C_o}{C_o} = \frac{\mu}{1 + \mu(r + \pi_o)} \tau e^{-rT}(\pi_o - \pi_1),
\]}

\[\frac{C_p - C_o}{C_o} = \frac{\mu}{1 + \mu(r + \pi_o)} \tau e^{-rT}(\pi_o - \pi_1),\]

where \(C_o\) is initial consumption; \(r\) is the world market real interest rate; \(\mu\) is the ratio of money balances to aggregate consumption (the parameter in the CIA constraint); \(\pi_o\) is the initial rate of crawl and \(\pi_1\) the rate during ERBS; \(T\) is the length of the ERBS program; and \(\tau\) is the intertemporal elasticity of substitution. For \(T = 3, r = .08, \mu = .10, \pi_o = 1\) and \(\pi_1 = .10\), we have \((C_p - C_o)/C_o = .064\tau\). The empirical evidence places \(\tau\) between .20 and .50 in LDCs (Agenor and Montiel, 1996, Table 10.1). But with \(\tau = .20 - .50\),}
the peak increase in consumption is only 1-3%. Consistent with this, Reinhart and Vegh (1995a) and Mendoza and Uribe (1996) found that the WC hypothesis predicts increases in consumption only 10-20% as large as the increases observed in the southern cone tablitas and other ERBS episodes.\(^3\) Thus both theory and empirical tests seem to argue that the WC hypothesis cannot deliver strong quantitative effects (Agenor and Montiel, 1996, p.353).

Since the Calvo-Vegh papers first appeared, theoretical research has moved on to investigate the properties of models with assorted wealth and supply-side effects in the hope of achieving a better fit with the stylized facts. This hope has not been borne out. After surveying the literature and conducting additional independent analysis, Rebelo and Vegh (1995) conclude that, even when combined for maximum impact, the proposed effects cannot account for the quantitative magnitude of the consumption boom and real exchange rate appreciation seen in ERBS programs. The bottom line in Uribe (2002, p.563), the most recent attempt to secure strong wealth/supply effects, is equally discouraging: “. . . existing models produce consumption booms and real exchange rate appreciations that are too small compared to the actual data. The quantitative analysis conducted in Section 6 . . . does not help resolve this problem.”

Repeated failure has taken a toll. New research on ERBS now has to battle uphill against the perception that “everything has been tried and nothing works.” This is unfortunate because the WC hypothesis was never given a fair hearing. Rebelo and Vegh (1995) and Reinhart and Vegh (1995) were careful to note that the spending boom might be much stronger in models that incorporate durable consumer goods. Certainly there is abundant casual evidence to support this conjecture. According to case studies and Calvo and Vegh’s (1999) stabilization time profiles, the boom-bust cycle is driven by the tremendous expansion and subsequent collapse in durables purchases. But despite the “hints” in the data, durables have not figured in most ERBS models. The sole exception is De Gregorio, Guidotti, and Vegh’s (1998) elegant analysis of the “bunching” pattern in durables spending when purchases follow a S-s rule. Their model, however, abstracts from nondurables consumption, treats wealth effects as largely exogenous, and assumes ERBS is permanent and fully credible. It is too stylized therefore to confront with the data or with the competing hypothesis that the consumption boom stems from weak credibility. The second limitation is partic-
ularly important, for the underlying theory does not establish a presumption that credible ERBS will trigger a boom-bust cycle in durables spending of exceptional amplitude and duration. In fact, there are good reasons to believe that, despite bunching, the assumption of perfect credibility is incompatible with large quantitative effects.\textsuperscript{4}

This paper reevaluates the WC hypothesis in a variety of models that accomodate both durable and nondurable consumer goods. We start with a simple one-sector model that permits the derivation of sharp analytical results. Many of the qualitative properties of the consumption path in the one-sector model depend on whether the elasticity of durables spending with respect to Tobin’s q ($\Omega$) is larger or smaller than the intertemporal elasticity of substitution ($\tau$). In the benchmark case of a separable utility function, we demonstrate that $\Omega > \tau$ is sufficient for: (i) durables spending to increase more than nondurables expenditure on impact; (ii) aggregate consumption to rise more than in the counterfactual scenario where all consumption is nondurable; (iii) durables spending to decrease more than nondurables consumption at the time of the policy reversal and (iv) durables spending to overshoot its lower steady-state level during the ERBS period. These results — especially overshooting — are consistent with durables being the most volatile component of aggregate consumption and with the stylized fact that durables spending leads in both the boom and the bust phases of the ERBS cycle.

The theoretical results are developed in the first two sections of the paper. In Section 3 we calibrate the model and present solutions for the global nonlinear saddle path. The numerical results confirm that weak credibility triggers a huge, double-digit spending boom. Aggregate consumption increases 9-18\% in the first year, rising to 12-28\% by the end of ERBS. Throughout, most of the heavy lifting is done by the smallest component of expenditure: durables comprise only 20\% of consumption but account for 70-90\% of the increase in total spending.

A satisfactory theory should explain not only the magnitude of the consumption boom but also the slope of the consumption path. The latter requirement has proven difficult (Uribe, 2002). Most ERBS models predict that, after an initial jump, the path of consumption is flat or declining during the low-crawl period. In reality, consumption either increases continuously or follows a hump-shaped path, with the downturn coming in the last 6-12
months of the program.

Our model predicts that the post-jump path of consumption is flat for the first half of the ERBS program and positively sloped in the second half. This improves on the existing literature but is still unsatisfactory. Accordingly, in Section 4 we investigate whether habit formation eliminates the problematic flat stretch in the consumption path. It does, albeit not for familiar, off-the-shelf specifications. The improvement in the results is minimal when habit enters the utility function in the normal way. But if habit affects durables spending (as opposed to the utility flow from consumption), the path of aggregate consumption is hump-shaped and the turning point comes at the right time, 6-9 months before ERBS collapses. Moreover, in contrast to the results in Uribe (2002), there is no marked tradeoff between the slope of the consumption path and its height. Depending on the specification, the peak increase in aggregate consumption is either about the same or twice as high as in the model without habit formation.

While the one-sector model is a useful device for thinking about the elements needed to explain the boom-bust cycle in consumption, it precludes analysis of other important stylized facts, most notably the impact of ERBS on the path of the real exchange rate. To remedy this, we add a nontradables sector in Sections 5-6 and compare outcomes in flexprice models with and without supply effects and for temporary/noncredible vs. permanent/credible programs. The results further strengthen the case for resurrecting the WC hypothesis. When there are no supply effects, the consumption boom peaks at 17-22% and the real exchange rate appreciates 24-26%. Augmenting the model with supply effects produces even better numbers. In our preferred specification, the consumption boom is equally strong but appreciation of the real exchange rate rises to 30-40%. In other runs, the consumption boom increases to 25-35% without diminishing appreciation of the real exchange rate (which stays in the 20-25% range). Supply effects are limited, however, to a secondary role. When stabilization is credible, their quantitative kick is bigger but insufficient to compensate for intertemporal substitution in durables spending. The consumption boom peaks at a modest 5% and the real exchange rate appreciates only 3-8%. Future research may overturn this conclusion, but, for now, the WC hypothesis stands alone as the only hypothesis that explains the stylized facts associated with ERBS.
1. A Simple One-Sector Model

The economy is small and completely open. Domestic output is fixed at $Q$ and the inflation rate equals the rate of crawl of the currency $\pi$. The private sector divides its wealth between money $m$ and a tradable bond $b$ that pays the world market interest rate $r$. $C$, $S$, and $D$ denote, respectively, nondurables consumption, gross new durables purchases, and the stock of durables.

We lay out the model in stages, starting with the specification of financial markets and the transactions technology.

Financial Markets and the Transactions Technology

Bonds are bought and sold in a perfect world capital market. The nominal interest rate $i$ is tied down therefore by the interest parity condition:

$$i = r + \pi. \quad (1)$$

Money is held to reduce transactions costs. These costs enter the budget constraint [see equation (4) below] via the term $(C + S)L[m/(C + S)]$, where $L$ is decreasing and strictly convex in the ratio of money balances to total spending ($L' < 0$, $L'' > 0$).

The Private Agent’s Optimization Problem

All economic decisions in the private sector are controlled by a representative agent who possesses an instantaneous utility function of the form $U(C, D) - R(\dot{D}/D)D$, where a dot signifies a time derivative and $U(\cdot)$ is increasing and strictly concave in $C$ and $D$. The $R(\cdot)D$ component of the utility function is taken from Bernanke (1985). It introduces a friction that prevents durables purchases from being absurdly volatile. As Bernanke emphasizes, new durables purchases are not easy or automatic: in contrast to spending on nondurables, the decision to buy a durable often involves time-consuming search and careful deliberation. The utility cost of worrying and lost leisure time is assumed to be increasing, symmetric, and convex in net purchases of durable goods: $R(0) = 0, R' \geq 0$ as $\dot{D} \geq 0$, and $R'' > 0$.

After imposing interest parity and defining $A \equiv m + b$ to be total wealth, the private
agent’s optimization problem may be written as

\[
\max_{C, S, m, b} \int_0^\infty \left[ U(C, D) - R \frac{S}{D} (S - \delta) \right] e^{-\rho t} dt,
\]

subject to

\[
\begin{align*}
A &= m + b, \\
\dot{A} &= Q + \tilde{g} + rb - (C + S) \left[ 1 + L \left( \frac{m}{C + S} \right) \right] - \pi m, \\
\dot{D} &= S - \delta D,
\end{align*}
\]

where \( \rho \) is the time preference rate; \( \tilde{g} = g + (C + S) L \) is lump-sum transfers; and \( \delta \) is the depreciation rate of of the durable good. Transfer payments are split into two components: government transfers, \( g \), and rebated profits of firms that supply transactions services, \( (C + S) L \). The artificial component, \( (C + S) L \), ensures that transactions costs wash out in the budget constraint. This eliminates a potentially dubious income effect. With the income effect removed, variations in the cost of liquidity influence spending only insofar as they alter the price of current vs. future consumption.

The Maximum Principle furnishes the necessary conditions for an optimum. These consist of

\[
\begin{align*}
U_C(C, D) &= \omega_1 (1 + L - L'm/X), \\
-L' &= \pi, \\
\omega_2 &= \omega_1 (1 + L - L'm/X) + R'(S/D - \delta), \\
\dot{\omega}_1 &= \omega_1 (\rho - r) = 0 \quad \text{for} \quad \rho = r, \\
\dot{\omega}_2 &= \omega_2 (\rho + \delta) + R - R'S/D - U_D,
\end{align*}
\]

where \( X \equiv C + S \) and \( \omega_1 \) and \( \omega_2 \) are the multipliers attached to the constraints (4) and (5). Equation (6) says that the marginal utility of nondurables consumption equals the shadow price of wealth multiplied by the effective price of consumption \( (1 + L - L'm/X) \), while (7) requires money to earn the same return at the margin as bonds. In equation (9) we have assumed \( \rho = r \) in order to abstract from trends in saving. Finally, equations (8) and (10) define a Tobin’s q model of durables purchases in which \( \omega_2/\omega_1 (1 + L - L'm/X) = \omega_2/U_C \) is
the ratio of the demand price (or shadow price) of a durable to its supply price (unity) and $R'$ captures additional adjustment costs incurred by increasing $S$ a small amount.

Path of the Crawl

The path for the crawl is

$$\pi = \begin{cases} 
\pi_1 < \pi_o & \text{for } 0 < t < T \\
\pi_o & \text{for } t > T 
\end{cases}$$  \hspace{1cm} \text{(11)}$$

ERBS commences with an announcement that the rate of crawl will be reduced from $\pi_o$ to $\pi_1$ and maintained at the lower level forever more. This proves false. Forever more lasts only until year $T$, at which time the government aborts the program and raises the crawl to its original level.

The Public Sector Budget Constraint

Money is injected into the economy whenever the central bank accumulates foreign exchange reserves $k$ or runs the printing press to finance the fiscal deficit. Assuming reserves are invested in the tradable bond, the consolidated public sector budget constraint reads

$$\dot{k} = rk + \pi m + \dot{m} - g.$$ \hspace{1cm} \text{(12)}$$

Fiscal policy is passive. Crucially, the reduction in the crawl is not supported by a cut in real transfer payments.\(^6\) When the program fails and $\pi$ returns to its original level, $g$ adjusts only enough to offset any changes in the sum of interest income and revenue from the inflation tax $(rk + \pi m)$.\(^7\)

Net Foreign Asset Accumulation and the Current Account Balance

Summing the public and private budget constraints produces the accounting identity that net foreign asset accumulation equals the current account surplus, viz.:

$$\dot{Z} = Q + rZ - C - S,$$ \hspace{1cm} \text{(13)}$$

where $Z \equiv k + b$. 

7
Functional Forms

To obtain concrete analytical results and prepare the model for calibration, we assume
\[
U(C, D) = \left\{ \frac{[a_1 C^{(\sigma-1)/\sigma} + a_2 D^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)}}{1 - 1/\tau} \right\},
\]
\[
R(S/D - \delta) = x(S/D - \delta)^2, \quad x > 0,
\]
\[
L \left( \frac{m}{C + S} \right) = h \left( \frac{m}{C + S} \right)^{1-1/\beta}, \quad h > 0, \quad 0 < \beta < 1,
\]
where \(a_1\) and \(a_2\) are distribution parameters, \(\sigma\) is the elasticity of substitution between durable and nondurable consumer goods, and \(\tau\) is the intertemporal elasticity of substitution. These are familiar functional forms. Nondurable consumption and the service flow from durables combine in a CES-CRRA function, while deliberation costs are a quadratic function of new durables purchases. The specification of transactions costs is the same as in Reinhart and Vegh (1995) and Uribe (2002).

The CES-CRRA utility function is flexible but a bit ungainly. In what follows, it will prove helpful to have some elasticity formulas at hand:
\[
-\frac{U_C}{U_{CC} C} = \frac{\sigma \tau}{\tau \theta_d + \sigma \theta_c}, \quad \frac{U_{CD} D}{U_C} = \frac{\tau - \sigma}{\tau \sigma} \theta_d,
\]
\[
-\frac{U_D}{U_{DD} D} = \frac{\sigma \tau}{\tau \theta_c + \sigma \theta_d}, \quad \frac{U_{DC} C}{U_D} = \frac{\tau - \sigma}{\tau \sigma} \theta_c.
\]
\(\theta_c\) and \(\theta_d\) are the respective shares of nondurables and durables in total consumption. It is easy to show from the first-order conditions that
\[
\theta_c = \frac{a_1 C^{(\sigma-1)/\sigma}}{a_1 C^{(\sigma-1)/\sigma} + a_2 D^{(\sigma-1)/\sigma}}, \quad \frac{U_C C}{U_C C + U_D D} = \frac{C}{C + (r + \delta)D},
\]
\[
\theta_d = \frac{a_2 D^{(\sigma-1)/\sigma}}{a_1 C^{(\sigma-1)/\sigma} + a_2 D^{(\sigma-1)/\sigma}} = 1 - \theta_c,
\]
evaluated at a steady state.
2. Solving the Model for Small Changes

It is possible to derive analytical results when the reduction in the crawl is small. This is worth doing. Although the final word rests with the solution for the global nonlinear saddle path, a lot can be learned about the general nature of the dynamics by solving the model for differential changes.

We start by manipulating the first-order conditions. Differentiate (8) with respect to time and substitute for \( \dot{\omega}_2 \) from (10). Since \( m/X \) and \( U_C \) are constant during intervals where \( \pi \) is constant, we get

\[
\dot{\omega}_2 = R'' \left( \frac{\dot{S}}{D} - \frac{S}{D^2} \dot{D} \right),
\]

\[
\Rightarrow \frac{R''}{D} \dot{S} = (\rho + \delta)U_C + (\rho + \delta - S/D)R' + \frac{R''S}{D^2} (S - \delta D) + R - U_D. \tag{14}
\]

Equations (5) and (14) are a self-contained sub-system in \( S \) and \( D \) when the utility function is separable. (\( U_C \) is constant in the first term.) In the non-separable case, however, we need to know how \( C \) varies on the transition path. This information is supplied by (6):

\[
dC = -\frac{U_{CD}}{U_{CC}} dD. \tag{15}
\]

Linearizing (14) around a stationary equilibrium \((\bar{S}, \bar{D})\) now produces

\[
\frac{R''}{D} \dot{S} = (\rho + \delta) \frac{R''}{D} [(S - \bar{S}) - \delta(D - \bar{D})] + \frac{U_{CD}^2}{U_{CC}} - U_{DD})(D - \bar{D}),
\]

or

\[
\frac{R''}{D} \dot{S} = (\rho + \delta) \frac{R''}{D} [(S - \bar{S}) - \delta(D - \bar{D})] - \frac{(\rho + \delta)U_C}{(\tau_{\theta_d} + \sigma_{\theta_c})D} (D - \bar{D}), \tag{16}
\]

after using the elasticity formulas. To relate \( R'' \) to observable magnitudes, write (8) as

\[
1 + R'(S/D - \delta)/U_C = q, \tag{17}
\]

where \( q \equiv \omega_2/U_C \) is Tobin’s \( q \), the ratio of the demand price of a durable to its supply price.
Differentiating with respect to $S$ and $q$ yields
\[
\frac{R'' S}{U_C D} \frac{dS}{S} = \frac{dq}{q}.
\]
Define $\Omega \equiv (dS/dq)q/S$ to be the elasticity of durables spending with respect to Tobin’s $q$. Evaluated at a steady state where $S/D = \delta$ and $q = 1$,
\[
R'' = \frac{U_C}{\Omega \delta}.
\]
The linearized system is thus
\[
\begin{bmatrix}
\dot{S} \\
\dot{D}
\end{bmatrix} =
\begin{bmatrix}
\rho + \delta & - (\rho + \delta)\delta - c \\
1 & -\delta
\end{bmatrix}
\begin{bmatrix}
S - \bar{S} \\
D - \bar{D}
\end{bmatrix},
\]
where
\[
c \equiv -\frac{(\rho + \delta)\delta \Omega}{\tau \theta_d + \sigma \theta_c} < 0.
\]
The steady state is a saddle point with eigenvalues
\[
\lambda_{1,2} = \frac{\rho \pm \sqrt{\rho^2 - 4c}}{2}, \quad \lambda_1 > 0, \; \lambda_2 < 0.
\]
During the ERBS phase, the dynamics are governed by a nonconvergent path of the system associated with the low rate of crawl $\pi_1$. For this phase, (18) gives
\[
S(t) - S_1 = (\lambda_1 + \delta)h_1 e^{\lambda_1 t} + (\lambda_2 + \delta)h_2 e^{\lambda_2 t}, \quad t < T, \quad (19)
\]
\[
D(t) - D_1 = h_1 e^{\lambda_1 t} + h_2 e^{\lambda_2 t}, \quad t < T, \quad (20)
\]
where $h_1$ and $h_2$ are constants and $(S_1, D_1)$ is the stationary equilibrium paired with $\pi_1$.
After the policy reversal at time $T$, the economy follows the saddle path that leads to the new long-run equilibrium $(S_2, D_2)$. On the convergent path, the term involving the positive eigenvalue drops out:
\[
S(t) - S_2 = (\lambda_2 + \delta)h_3 e^{\lambda_2 t}, \quad t \geq T, \quad (21)
\]
\[
D(t) - D_2 = h_3 e^{\lambda_2 t}, \quad t \geq T. \quad (22)
\]
$h_3$ is another constant. Also, we have exploited the fact that for small changes the negative eigenvalue is the same as in (19) and (20).

Turning back to (15), interpret the differentials as deviations from the stationary equilibrium and write the path of $C$ as

$$C(t) = C_i - \frac{U_{CD}}{U_{CC}}[D(t) - D_i],$$

$$\implies C(t) = C_i + f[D(t) - D_i],$$

(23)

where

$$f \equiv \frac{(\tau - \sigma)\theta_c(\rho + \delta)}{\tau\theta_d + \sigma\theta_c}.$$  

Across steady states, $U_D/U_C = \rho + \delta$. This and homothetic preferences imply

$$C_i - C_o = \frac{C_o}{D_o}(D_i - D_o), \text{ } i = 1, 2.$$  

(24)

There are two more steps in the solution procedure: (1) derive the five boundary conditions that pin down $D_1$, $D_2$, and $h_1$- $h_3$ ($S_i = \delta D_i$ takes care of durables spending); (2) plug the solutions into (19)-(23) and extract conditions that delineate the path of spending. Both steps involve a good deal of tedious algebra. In a supplementary appendix (available upon request), we show that

$$D_1 > D_o > D_2,$$

$$\hat{S}(0), \hat{C}(0) > 0, \text{ with } \hat{S}(0) > \hat{C}(0) \text{ if }$$

$$\Omega > \frac{\rho\theta_c(\tau - \sigma) + \delta\tau}{\delta(\tau\theta_d + \sigma\theta_c)},$$

(25a)

$$\Omega > \sigma \left[1 + \frac{\delta(\sigma - \tau)\theta_d}{(\rho + \delta)(\tau\theta_d + \sigma\theta_c)}\right],$$

(25b)

$$\hat{S}(T), \hat{C}(T) < 0 \text{ with } |\hat{S}(T)| > |\hat{C}(T)| \text{ iff } \Omega > \frac{\sigma\tau}{\tau\theta_d + \sigma\theta_c},$$

(25)

$$S(t) < S_2 \text{ iff } \lambda_2 + \delta < 0$$

$$\implies S(t) < S_2 \text{ iff } \Omega > \tau\theta_d + \sigma\theta_c.$$  

(27)
The conditions in (25)-(27) have the same general structure: in each, $\Omega$ stands alone on the left side and a term involving $\sigma$ and $\tau$ occupies the right. These parameters claim the spotlight because they play pivotal roles in conditioning the intertemporal responses of $S$ and $C$. The scope for intertemporal substitution in nondurables consumption is tied to concavity of the utility function in $C$, which depends on both $\sigma$ and $\tau$. Concavity of the utility function also affects durables spending but is much less important as very little of the durable good is consumed at the time of purchase. Intertemporal substitution is limited instead by rising marginal deliberation costs. This friction is subsumed in the elasticity of $S$ with respect to Tobin’s $q$. Consequently, when $\Omega$ is large relative to $\sigma$ and $\tau$, the response of durables to intertemporal variations in the effective price of consumption is stronger than the response of nondurables. In the subsections that follow, we will be precise about what “large relative to $\sigma$ and $\tau$” means.

2.1 The Benchmark Case of a Separable Utility Function ($\sigma = \tau$)

Econometricians have yet to estimate the value of $\sigma$ for even one LDC. Nevertheless, there is some basis for the view that a separable utility function deserves the status of the benchmark case. Estimates of demand systems with 5-10 goods find that compensated own-price elasticities in both developed and less developed countries are on the order of .15-.65 (Lluch et al., 1977; Deaton and Muellbauer, 1980; Blundell et al., 1993), implying a slightly higher range of .20-.75 for the intratemporal elasticity of substitution. Some of the product categories in the demand systems refer mainly to semi-durables (e.g., clothing and textiles); others mix durable, non-durable, and semi-durable goods. At present, therefore, the best educated guess is that $\sigma$ also lies in the .20-.75 range. Since this closely overlaps the estimated range for $\tau$ in LDCs, we center the analysis around the dynamics for $\sigma = \tau$.

The results in the benchmark case are exceptionally clean. When the utility function is separable, the conditions in (25a),(25b),(26) and (27) all reduce to $\Omega > \tau$. This gives

**Proposition 1** When the utility function is separable between durables and nondurables, $\Omega > \tau$ is necessary and sufficient for durables spending to (i) increase more than nondurables spending at the start of ERBS [$\dot{S}(0) > \dot{C}(0)$], (ii) decrease more than nondurables spending at the time of the policy reversal, and (iii) overshoot and remain below its steady-state level throughout the post-ERBS period [$S(t) < S_2$, $t > T$].

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Figure 1 shows the complete transition path. The $\dot{S} = 0$ and $\dot{D} = 0$ schedules determine which way the north-south and east-west directional arrows point during the ERBS phase, while $A_i A_i$ is the saddle path associated with $(S_i, D_i)$. In the lower quadrant, we use equation (23) to track the path of nondurables consumption.

When ERBS is announced, $C$ jumps to $C_1$ and $S$ jumps to a point above $A_1 A_1$ (see the appendix). After the initial jumps, $C$ stays at $C_1$ until ERBS collapses while $S$ either rises continuously or follows a shallow U-shaped path. In the latter case, the transition path includes a period of decreasing expenditure. This phase is strictly transitory. Before the program fails, the path crosses the $\dot{S} = 0$ schedule. In the numerical simulations, the crossing point is typically around the middle of the second year (when $T = 3$).

The consumption spree and ERBS end at the same moment. When the crawl abruptly increases at $T$, spending plummets across-the-board. The diagram suggests and the numerical simulations will shortly confirm that the collapse is concentrated in durables purchases: $C$ jumps to $C_2$ and is constant thereafter, but $S$ overshoots $S_2$. Durables lead in the sudden crash just as they lead in the boom. Over the whole cycle, the gyrations in aggregate consumption mirror the volatile dynamics of durables expenditure.

### 2.2 Edgeworth Substitutes ($\sigma > \tau$)

In the case where durables and nondurables are Edgeworth substitutes, $\Omega \geq \sigma$ guarantees the conditions in (25a), (26), and (27), but not the condition in (25b). This can be handled by pushing $\Omega$ a little above $\sigma$. Write (25b) as

$$\Omega > \sigma \left[ 1 + \frac{(\sigma - \tau)(1 - \gamma)}{\tau(1 + \rho/\delta)(1 - \gamma) + \sigma \gamma} \right],$$

where $\gamma \equiv C/(C + \delta D)$ is the share of nondurables in total consumption spending. Since the term in square brackets is less than $1/\gamma$, we have

**Proposition 2** When durables and nondurables are Edgeworth substitutes, $\Omega > \sigma/\gamma$ is sufficient for durables spending to (i) increase more than nondurables spending at the start of ERBS [$\dot{S}(0) > \dot{C}(0)$], (ii) decrease more than nondurables spending at the time of the policy reversal, and (iii) overshoot and remain below its steady-state level throughout the post-ERBS period [$S(t) < S_2, t > T$].

**Remark 1** The data place $\gamma$ around .80 in LDCs; hence $\Omega$ does not have to be very much above $\sigma$ to meet the sufficient condition.
Figure 2 portrays the path to the new steady state. The main difference compared to the benchmark case is that both components of consumption overshoot at time $T$.

### 2.3 Edgeworth Complements ($\tau > \sigma$)

The conditions in (25a) and (25b) are jointly sufficient for $\hat{S}(0) > \hat{C}(0)$. To derive the strongest result for the case of Edgeworth complements, we also need to consult the necessary and sufficient condition

$$\hat{S}(0) > \hat{C}(0) \iff \Omega x_2(1 - e^{-\lambda_2 T}) > \left(\frac{\sigma \tau}{\tau \theta_d + \sigma \theta_c} - \Omega\right) \left[x_2 \frac{r(1 - \gamma)}{\delta} + e^{-\lambda_2 T}\right], \quad (28)$$

where

$$x_2 = 1 + \frac{\lambda_2 \tau}{\delta \Omega} \text{ sign } \Omega - \frac{\tau \rho \theta_c (\tau - \sigma) + \delta \tau}{\delta (\tau \theta_d + \sigma \theta_c)}. \quad \text{Positive when (25a) holds}$$

The term in square brackets is always positive (see the appendix). Thus the above condition holds for $x_2 < 0$ and $\Omega > \sigma \tau / (\tau \theta_d + \sigma \theta_c)$. On the other hand, if $x_2$ is positive, then (25a) must hold. We need only appeal to (25b) therefore to ensure $\hat{S}(0) > \hat{C}(0)$. But since $\tau > \sigma$ in the case under consideration, any value of $\Omega$ that satisfies (25a) also satisfies the weaker condition (25b). The upshot of all this is

**Proposition 3** When durables and nondurables are Edgeworth complements,

$$\Omega > \text{Max}\{\sigma \tau / (\tau \theta_d + \sigma \theta_c), \tau \theta_d + \sigma \theta_c\}$$

is sufficient for durables spending to (i) increase more than nondurables spending at the start of ERBS [$\hat{S}(0) > \hat{C}(0)$], (ii) decrease more than nondurables spending at the time of the policy reversal, and (iii) overshoot and remain below its steady-state level throughout the post-ERBS period [$S(t) < S_2, \ t > T$].

**Remark 2** The borderline value of $\Omega$ that satisfies the sufficient condition is smaller than $\tau$. ($\tau > \sigma$ and the arguments of $\text{Max}\{\cdot\}$ are the weighted harmonic mean and the weighted arithmetic mean.)

Edgeworth complementarity changes the look of the adjustment process in two ways (Figure 3). First, nondurables consumption rises throughout the ERBS phase and undershoots its steady-state level in the post-ERBS period. Second, $S$ does not necessarily jump to a
point above the $A_1A_1$ schedule (see the appendix). Consequently, durables spending may decrease steadily after its initial jump.

### 2.4 The Grand Corollary

Absent information about the sign of $\sigma - \tau$, it is useful to have a general result that applies irrespective of whether durables and nondurables are Edgeworth substitutes, Edgeworth complements, or independent. The union of Propositions 1-3 fills the bill:

**Corollary 1** $\Omega > \text{Max}\{\sigma/\gamma, \tau\}$ is sufficient for durables spending to (i) increase more than nondurables spending at the start of ERBS [$\hat{S}(0) > \hat{C}(0)$], (ii) decrease more than nondurables spending at the time of the policy reversal, and (iii) overshoot and remain below its steady-state level throughout the post-ERBS period [$S(t) < S_2, t > t_1$].

Taking stock, after three propositions and one corollary, where do we stand? Certainly the weak credibility hypothesis has regained some of its swagger. Given the weight of the evidence that $\sigma$ and $\tau$ are well below unity and the tremendous volatility of durables spending in ERBS programs, there is not much doubt that $\Omega$ is considerably larger than both $\sigma/\gamma$ and $\tau$ and that the boom-bust cycle in durables expenditure drives the boom-bust cycle in aggregate consumption. On its own, however, this is not enough to rehabilitate the hypothesis. The analytical results were derived for small changes. As such, they are only suggestive of strong quantitative effects. Confirmation is needed from numerical simulations that the predicted paths for durables expenditure and aggregate consumption lie within shouting distance of the big numbers seen in the data.

### 3. Calibration of the Model and Numerical Results

To calibrate the model, we set

$$\gamma_o = .80, \quad \mu_o = .10, \quad \beta = .50, \quad \pi_o = 1, \quad \pi_1 = 0, \quad \delta = .10, \quad T = 3, \quad r = .05,$$

and let $\sigma$, $\Omega$ and $\tau$ assume multiple values:

$$\sigma = .25, .50, .75, \quad \tau = .25, .50, \quad \Omega = 5, 10.$$

The ratio of money balances to aggregate spending ($\mu$) is 10%. This is a conservative choice.
closer to the ratio of money to GDP than to the ratio of money to private consumption. It is intended to counteract any bias toward a strong consumption boom caused by the absence of a nontradables sector. With respect to the other choices:

- **Elasticity of money demand with respect to the interest rate** ($\beta$). Reinhart and Vegh (1995a), Rossi (1989), and Arrau et al. (1995) have estimated money demand functions of the type employed in our model. The value assigned to $\beta$ is almost the same as the average of their estimates for Argentina, Brazil, Mexico, Chile and Uruguay.\(^{11}\)

- **Consumption share of durables** ($1 - \gamma$). The share of durables in aggregate spending is $S/(C + S)$. A figure close to this can be computed from the United Nations National Income Accounts. For a broad definition that includes semi-durables, the share lies in the .18-.22 range: .223 for Mexico (2000), .179 for Colombia (1998), .180 for Bolivia (1992), .203 for the Philippines, and .217 for S. Africa (2001) and S. Korea (2002). The value in the model (.20) is the average of the values for Mexico and Colombia.

- **Depreciation rate for durables** ($\delta$). The Central Statistical Office of Great Britain and the U.S. Department of Commerce estimate the service life to be ten years for major appliances, cars, and other vehicles (Williams, 1998). We used this figure to fix $\delta$. (There are no data for LDCs.)

- **Length of the ERBS program** ($T$). The low-crawl period lasts three years. Three is a popular choice in the literature and close to the average value in Calvo and Vegh’s (1999) dataset for major ERBS episodes.

- **Rate of crawl before vs. during ERBS** ($\pi_0, \pi_1$). The numerical simulations cut the rate of crawl from an initial value of 100% to zero during ERBS. This is larger than the reductions in the Chilean and Uruguayan tablitas but far smaller than the reductions in the Argentine tablita, Mexico’s Solidarity Pact, or Argentina’s Convertibility Plan.

- **Real interest rate** ($r$). Reinhart and Vegh (1995a) peg the world market real interest rate at 3%. Burstein et al. (2001) and Rebelo and Vegh (1995) opt for 4.1%, while Uribe (2002) prefers 6.5%. We compromise on 5%, a value inbetween the long-run real returns paid by U.S. stocks and treasury bonds.

- **Intertemporal elasticity of substitution** ($\tau$). The low and high values for the intertemporal elasticity of substitution are in line with estimates for LDCs, most of which place $\tau$ somewhere between .20 and .50 (Agenor and Montiel, 1996, Table 10.1).

- **Elasticity of substitution between durable and nondurable consumer goods** ($\sigma$). Although there are no estimates of $\sigma$ for LDCs, a range of .25-.75 is at least consistent with the empirical evidence that compensated elasticities of demand tend to be small at high levels of aggregation. This range is also wide enough to encompass cases of Edgeworth complementarity and Edgeworth substitutability.

- **Elasticity of durables spending with respect to Tobin’s q** ($\Omega$). Baxter (1996) sets $\Omega$ at 200. (Note: 200 is not a typo.) Baxter and Crucini (1993) and Rebelo and Vegh (1995) choose
15 in models where the durable good is physical capital. 200 and 15 appear to be a pure
guesses. (Our own literature search for information about \( \Omega \) proved fruitless.) We restrict
\( \Omega \) to much lower values but still get tremendous variation in durables expenditure over
the ERBS cycle.

3.1 Solution Technique

The numerical solutions report the results for the global nonlinear saddle path. This required
a technical innovation to deal with the unit root problem generic to models that postulate
an exogenous world market interest rate. (In continuous time, the unit root shows up as
a zero eigenvalue.) Due to the unit root, it is not possible to solve for the steady state
independent of the transition path. But without prior knowledge of the steady state, a
conventional shooting program does not know what to shoot for. The existing literature has
relied therefore on linear approximations to the true solution.

We solved the unit root problem by employing a different search strategy. Instead of
shooting for the path that takes the economy to the unknown steady state, our program
shoots for the path that eventually satisfies the conditions for a stationary equilibrium.
Guided by this strategy, the program solves for the transition path and the steady state
simultaneously. See Atolia and Buffie (2007) for a more detailed discussion of the algorithm.

3.2 Numerical Results

Figures 4-7 show the percentage deviations of nondurables consumption, durables spending,
and aggregate consumption \( (C + S) \) from their initial values. There are four runs with a
separable utility function and two each for the cases of Edgeworth complementarity and
Edgeworth substitutability. In the figure for aggregate consumption, the dashed line is the
path when all consumption is nondurable. The vertical distance between this line and the
actual path reflects the impact of durables expenditure at each point in the cycle.

What stands out in a quick pass through the figures are the big numbers for aggregate
consumption and durables spending. The peak increase ranges from 12\% to 27\% for ag-
gregate consumption and from 46\% to 117\% for durables. Furthermore, the “fit” between
the numerical results and the stylized facts pertaining to the composition of consumption
growth and the volatility of durables expenditure is remarkably good. In the panel regres-
sions reported by DeGregorio et al. (1998), annual growth of durables spending averages
21% for the first three years, with the 95% confidence interval bracketing 8.5%-33.7%.\textsuperscript{12} The point estimate of 21% is 3.5 times larger than the estimate for total consumption growth. By comparison, in the eight cases covered by Figures 4-7, the peak increase in durables spending is 3-5 times larger than the peak increase in aggregate consumption and the average growth rate for durables ranges from 13% to 29%. The numbers are also close for the bust phase of the cycle. DeGregorio et al.’s alternative point estimates have durables expenditure plunging 21-72% (relative to the pre-ERBS trend line) in the year after ERBS collapses. The range in our simulations is -31% to -75%.

4. Habit Formation

While the model delivers good results for the magnitude and the composition of the consumption boom, it does less well in accounting for the slope of the consumption path. The time profiles of aggregate consumption and durables spending are essentially the same in each case: a sharp jump at $t = 0$, a succeeding flat stretch for the next 1.5 years, and then rapid, accelerating growth until the end of year three. This is at variance with the facts. Consumption rose continuously in some ERBS episodes (e.g., Argentina, 1967-1970; Brazil, 1964-1968; Mexico, 1988-1994; Peru, 1985-1987; Paraguay, 1991-1997);\textsuperscript{13} in others, a flat stretch or a downturn materialized, but only in the last year of the program.

The results in Uribe (2002) suggest that a model with habit formation might generate a more realistic path for consumption. Following Carroll et al. (2000) and Fuhrer (2000), let the stock of habit $H$ grow at the rate

$$\dot{H} = v(N - H),$$

where $v > 0$ and

$$N \equiv [a_1C^{(\sigma-1)/\sigma} + a_2D^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)}.$$

Habit enters the utility function with the persistence parameter $\alpha$. Two formulations dominate the literature:

$$U(N, H) = \frac{(N - \alpha H)^{1-1/\tau}}{1 - 1/\tau}, \quad 0 < \alpha < 1$$

(30a)
and
\[ U(N, H) = \frac{(N/H^\alpha)^{1-1/\tau}}{1 - 1/\tau}, \quad 0 < \alpha < 1. \] (30b)
In the additive specification, utility is a function of the difference between current consumption and the stock of habit (also called the “subsistence level of consumption”). The multiplicative specification assumes that felicity depends instead on consumption relative to the stock of habit. Both specifications capture the basic idea of habit formation — that the private agent desires to smooth the change as well as the level of consumption.

There is general agreement in the literature that the persistence parameter \( \alpha \) is on the order of .70. Unfortunately, no similar consensus exists about the likely value of \( \nu \). Numerous papers postulate very rapid habit formation, citing Fuhrer (2000) for support. It is common practice, for example, to set the stock of habit equal to last quarter’s consumption. At the opposite extreme, the evidence marshalled in Constantinides (1991), Heaton (1995), and Boldrin et al. (1997) suggests that it takes years for habits to fully adjust. Carroll et al. (2000) and Mansoorian and Michelis (2005) subscribe to this view, calibrating their models with \( \nu = .20 \).

In the next two sections we present results for the multiplicative specification when \( \tau = \sigma = .25, \Omega = 10, \alpha = .70, \) and \( \nu = .5, 3 \) or 6.5. In the runs with \( \nu = 6.5 \), the stock of habit covers 80% of the distance to its new long-run level within one quarter. For \( \nu = .5 \), the 80% point is not reached until 3.25 years elapse.

4.1 Numerical Results

Habit formation moderates consumption growth most in the early and late stages of ERBS as the private agent tries to smooth the large jumps in \( C \) and \( S \) at \( t = 0 \) and \( t = T \). Beyond this, not much can be said. It is not clear a priori how the peak and the slope of the spending path will change compared to the benchmark model. A closer look at the utility function in (30b) shows why. In the extreme short run (where \( H \) is fixed), the intertemporal elasticity equals \( \tau \) and the utility function is separable in durables and nondurables consumption \( (U_{CD} = 0) \). Over time, however, intertemporal substitution becomes easier (Carroll et al., 2000) and accumulation of durables exerts a positive effect on the marginal utility of
nondurables consumption. After habits have fully adjusted,

\[-\frac{U_N}{U_{NN}N}_{dH=dN} \equiv \tilde{\tau} = \frac{\tau}{\tau\alpha + 1 - \alpha} > \tau \text{ for } \tau < 1\]

and

\[\frac{U_{CD}D}{U_C}_{dH=dN} \equiv \tilde{\eta} = \frac{\theta_d\alpha(1 - \tau)}{\tau} > 0 \text{ for } \tau < 1.\]

Since \(\tilde{\tau} > \tau\), the consumption boom may be stronger than in the model without habit formation. This is most likely when \(v = 6.5\) and the intertemporal elasticity of substitution rises quickly to \(\tilde{\tau}\).

The relationship between habit formation and the trajectory of nondurables consumption is more complex. As usual, higher consumption today raises the marginal utility of consumption tomorrow by increasing the stock of habit. Growth in the stock of habit also increases the degree of complementarity between durables and nondurables (\(\tilde{\eta}\) rises with \(H\)). If the story ended here, we could be sure that habit formation would impart an upward tilt to the path of nondurables consumption. But it doesn’t. There is a complicating factor: the desire to smooth the fall in consumption at \(T\) tends to ratchet the consumption path southward immediately after the jump at \(t = 0\).

Figure 8 confirms these analytic-based conjectures. The results for \(v = .50\) in Panel A are disappointing. Raising \(v\) to 6.5, however, gives rise to a nice hump-shaped path for nondurables consumption and a generally better fit with the stylized facts. Absent habit formation, nondurables consumption jumps 3.4% at \(t = 0\) and then stays flat for the rest of the ERBS period. In Panel B, \(C\) surges 6% in the first six months, rising to a peak of 8% at \(t = 2.25\). Since durables spending also rises more at the outset, total consumption growth at \(t = 1\) and \(t = 2\) is 5-6 percentage points higher than in the model without habit formation. The gap narrows in the third year but does not disappear. Thus the time profiles for aggregate consumption and its principal components are uniformly superior to the time profiles in the model without habit formation. Unlike in Uribe (2002), there is no tradeoff of lower height for better slope — the path of aggregate consumption is continuously above the no habit formation path.

Two additional points merit comment. First, regardless of whether one favors the multi-
plicative or the additive specification, the case for habit formation requires very fast adjustment in the stock of habit. There is not much to choose from between a model with slow or moderately fast habit formation and a model with no habit formation. Second, while the introduction of rapid habit formation produces better results, it is not a complete fix. The paths for durables spending and aggregate consumption still have the wrong shape — growth should be greater in the second year than in the third, not the other way around. This motivates us to investigate a less conventional specification of habit formation.

4.2 Habit Formation in Durables Spending

So far we have followed Bernanke (1985) in assuming that deliberation costs depend on how fast the stock of durables changes. This is not the only sensible specification. It is equally plausible that the private agent experiences psychological unease when durables spending \( S \) varies from its customary level. Suppose therefore

\[
R(S, H) = x \frac{(S/H - 1)^2}{2} H, \tag{31}
\]

where

\[
\dot{H} = v(S - H), \quad v > 0. \tag{32}
\]

Naturally, we hope that habit formation will now generate a smooth hump-shaped path for \( S \) and, by extension, for aggregate consumption. The idea is not entirely new. Burnside et al. (2004) have employed a similar strategy to dampen the volatility of investment in a real business cycle model.

Panel A in Figure 9 shows the outcome when \( \tau = \sigma = .25, \Omega = 5, v = 3, \) and habit formation appears only in the deliberation cost function. We would like to include habit formation in the utility function as well, but, at present, our computer program cannot solve for the nonlinear saddle path in systems that have more than three jump variables. The new specification is \textit{perforce} less than ideal.

Even so, the run in Panel A gives us almost everything we want. The paths for durables spending and aggregate consumption are hump-shaped, with expenditure contracting rapidly in the last six months of the program. Equally important, the pace of consumption growth does not slow until the start of the third year. Durables spending increases 46% in year
one, 42% in year two, and 11% in the first half of year three. The corresponding numbers for aggregate consumption are 12%, 12% and 5%. Perhaps the most surprising result is that the peak increases in durables spending and aggregate consumption are 2-2.5 times as large as in the model without habit formation. This is a natural consequence, however, of substituting $H$ for $D$ in the deliberation cost function. Increases in $H$ reduce marginal deliberation costs in (47) in the same way that increases in $D$ do when $R = x(S/D - \delta)^2 D$. But habit formation is extremely fast relative to durables formation ($v$ is large and $H$ is only a tenth the size of $D$). Hence, after about six months, the higher level of $S$ becomes routine and marginal deliberation costs decrease sharply. This paves the way for a boom that is both smoother and much more powerful than in the model without habit formation.

**Sensitivity Analysis**

The numerical simulations in Panel A assume that all durables spending enters the scale variable in the liquidity cost function. If some durables are pure credit goods, this exaggerates the incentive to shift purchases from the future to the present when the reduction in the crawl is temporary. Our own view is that most durables spending belongs in the liquidity cost function. Given the paucity of empirical evidence, however, the right specification of the scale variable is a judgment call.

We have undertaken additional runs to test the sensitivity of the results to the coefficient on $S$ in the liquidity cost function. In Panels B and C of Figure 9, we write the scale variable as $C + \xi S$ and reduce $\xi$ from unity to .50 or .25.

The results hold up surprisingly well. Because smaller values of $\xi$ lessen the impact of $S$ on liquidity costs, the percentage decrease in durables spending is much smaller than the decrease in $\xi$. The peak increase in aggregate consumption drops from 29% when $\xi = 1$ (Panel A) to 22% when $\xi = .50$ and to 18% when $\xi = .25$. These are nontrivial decreases, but 18% and 22% are still big numbers. Clearly, $\xi$ close to unity is not essential to a durables-driven explanation of the consumption boom.
5. Adding a Nontradables Sector

In this section we add a nontradables sector to the model. This allows us to demonstrate that the powerful consumption boom is not an artifact of the one-good model and that the WC hypothesis can explain strong, sustained appreciation of the real exchange rate.

The traded good, now called the export good, serves as the numeraire in the expanded model. Thus, unless otherwise indicated, prices and monetary aggregates are divided by the nominal exchange rate. Other notational conventions are as follows: \( P_n, w, Q_i, L_i, K_i, \) and \( C_i \) refer to the price of the nontraded good, the wage, production, employment and capital in sector \( i \) \((i = n,x)\), and nondurables consumption of good \( i \).

**Technology and Labor Demand**

Competitive firms operate CES production functions

\[
Q_x = F(K_x, L_x) = \left[ a_3 L_x^{(\sigma - 1)/\sigma} + a_4 K_x^{(\sigma - 1)/\sigma} \right]^{\sigma/(\sigma - 1)}, \tag{33a}
\]

\[
Q_n = G(K_n, L_n) = \left[ a_5 L_n^{(\sigma - 1)/\sigma} + a_6 K_n^{(\sigma - 1)/\sigma} \right]^{\sigma/(\sigma - 1)}, \tag{33b}
\]

and hire labor up to the point where its marginal value product equals the wage:

\[
F_L = w, \tag{34}
\]

\[
P_n G_L = w. \tag{35}
\]

Labor is intersectorally mobile, but the capital stocks are fixed. The latter assumption will be relaxed when we introduce supply effects in Section 8.

Consumption of multiple durable goods greatly complicates the model. To avoid this, we assume that one unit of the imported durable always combines with \( a_7 \) units of the nontraded durable. The price of the composite durable is thus

\[
P_d = 1 + a_7 P_n. \tag{36}
\]

**The Private Agent’s Optimization Problem**

The private agent solves his optimization problem in two stages. In the first stage, \( C_n \) and \( C_x \) are chosen to maximize \( C(C_n, C_x) \), subject to \( P_n C_n + C_x = E \), where \( E \) is total.
nondurables expenditure and $C(C_n, C_x)$ is a linearly homogenous CES aggregator function. The optimal choices $\bar{C}_n$ and $\bar{C}_x$ are subsumed in the indirect utility function

$$V(P_n, E) = C[\bar{C}_n(P_n, E), \bar{C}_x(P_n, E)] = E/c(P_n),$$

where

$$c(P_n) = \left(k_o^\beta + k_1^\beta P_n^{1-\beta}\right)^{1/(1-\beta)},$$

$k_0$ and $k_1$ are distribution parameters, and $\beta$ is the elasticity of substitution between the two consumer goods.

In the second stage of optimization, the agent chooses $m, b, E,$ and $S$ to maximize

$$U = \int_0^\infty \left[ \frac{(N/H_1)^{1-1/\tau}}{1-1/\tau} - x \frac{(S/H_2 - 1)^2}{2} \right] e^{-\rho t} dt,$$

subject to

$$A = m + b,$$
$$\dot{A} = P_n Q_n + Q_x + \bar{g} + rb - (E + P_d S) \left[ 1 + L \left( \frac{m}{E + P_d S} \right) \right] - \chi m,$$
$$\dot{D} = S - \delta D,$$
$$\dot{H}_1 = v(N - H_1),$$
$$\dot{H}_2 = v(N - H_2),$$

where

$$N \equiv \{ a_1 E/c(P_n) \}^{(\sigma - 1)/\sigma} + a_2 D^{(\sigma - 1)/\sigma} \frac{1}{\sigma}.$$

We allow habit formation to operate in the utility function as well as the deliberation costs function. This maximizes the probability of getting realistic, hump-shaped paths for both durables spending and nondurables consumption.

**Net Foreign Asset Accumulation and the Current Account Balance**

Summing the budget constraints of the private agent and the government now yields

$$\dot{Z} = P_n Q_n + Q_x + rZ - E - P_d S.$$
Market-Clearing Conditions

The model is closed by the conditions that demand equal supply in the labor and non-tradables markets:

\[ L_x + L_n = \bar{L} \]  \hspace{1cm} (44)
\[ C_n + a_7 S = Q_n. \]  \hspace{1cm} (45)

The supply of labor is fixed at \( \bar{L} \). In (44), \( C_n \) is retrieved from the indirect utility function via Roy’s Identity:

\[ C_n = -\frac{\partial V}{\partial P_n} = E \frac{k_1 P_n^\beta}{k^\beta + k_1^\beta P_n^{1-\beta}}. \]  \hspace{1cm} (46)

Solution Technique

The core dynamic system in the model has four jump variables \([E, S, \text{ and the multipliers associated with (41) and (42)}]\) and four state variables \((D, Z, H_1, \text{and } H_2)\). Our algorithms cannot solve these higher-order systems for the global nonlinear saddle path. Consequently, the zero eigenvalue/unit root reappears as an insoluble problem (see the discussion in Section 4.1).

To circumvent this difficulty, we adopt Schmitt-Grohe and Uribe’s (2003) suggestion to let the real interest rate depend very weakly on the country’s debt-GDP ratio:

\[ r = \rho + \kappa \left( \frac{b}{P_n Q_n + Q_x} - \frac{b_o}{P_{n,o} Q_{n,o} + Q_{x,o}} \right), \quad \kappa < 0. \]  \hspace{1cm} (47)

\( f \) is assigned a small value to close to zero so that the loan supply curve is almost perfectly flat. This converts the zero eigenvalue in the perfect capital markets model into a tiny negative eigenvalue in the approximating model. It also makes the stationary equilibrium independent of the transition path. In the long run, \( r = \rho \) requires the foreign debt to change by the same percentage amount as the dollar value of real GDP. This and the other equilibrium conditions in the model tie down the steady state.

5.1 Calibration of the Model and Numerical Solutions

“At a quantitative level the results for our baseline parameterization fall short of explaining the orders of magnitude involved in stabilization episodes, suggesting that the
large consumption booms and the sizable real appreciations are puzzling.” (Rebelo and Vegh, 1995, p.168)

Table 1 lists the parameter values used to calibrate the expanded model. None of the values are terribly exotic. Durables are more import intensive than nondurables, habit formation is rapid \( (v = 6.5) \), and ordinary numbers are assigned to the cost share of labor, the elasticity of substitution between traded and nontraded nondurables, and the elasticity of substitution between capital and labor (all equal .50). The ratio of money balances to consumption is .15 vs. .10 in the one-sector model.\(^{17}\) The higher value is in line with the ratio of M1 to consumption for many ERBS episodes in Latin America. It is too low, however, for most episodes in Africa.\(^{18}\) Finally, the value of \( \kappa \), which fixes the slope of the loan supply curve in (47) is .0001. An increase in the debt-GDP ratio from 25% to 125% thus raises the borrowing rate by only one basis point.

Figure 10 shows the paths of aggregate consumption, durables spending, and the relative price of the nontraded good in the base run where \( \tau = .25 \) and \( \sigma = .75 \). The peak increases in these variables for all runs are collected in Table 2. This is done to save space and facilitate comparisons across models and parameter values. The graphs always have the same general shape; what differs is the scale on the vertical axis.

Despite its simplicity, the no-frills flexprice model performs quite well. The real exchange rate appreciates 24-26%, while the peak level of aggregate consumption ranges from 17% to 22%. As usual, the consumption boom is driven by a tremendous surge in durables spending. Observe also that adjustment is difficult in the post-ERBS period. In the first year after the policy reversal, the slump deepens and the real exchange rate depreciates continuously. This is followed by slow recovery to the pre-ERBS equilibrium. Even at year five, aggregate consumption and the real exchange rate are 8-10% below their initial levels.

6. Supply Effects and Temporary vs. Permanent ERBS

The stylized facts for successful ERBS programs look much like those for failed programs.\(^{19}\) This a problem for the preceding models. When ERBS is credible, the relative price of current consumption does not decrease and the private sector does not shift spending from the future to the present. Consequently, adjustment is instantaneous. There are no transitory
changes in consumption, the real exchange rate, or the current account deficit. The economy jumps immediately to the new low-inflation steady state.

In the case of a credible program, wealth effects have to propel the consumption boom. Most of the literature has focused on models where increases in wealth stem from supply-side expansion. As noted in the introduction, these models have not been successful in explaining the stylized facts. But this might change in more elaborate models that distinguish between nondurable and durables consumption. Durables spending responds strongly to wealth shocks. Even a modest increase in wealth has the potential therefore to trigger a multi-year consumption boom. The sticking point is that the normal definition of boom does not apply. When the context is ERBS, we need really big numbers.

6.1 One Last Model

“... further work on the structure of the supply-side and on the differential response of the tradable and non-tradable goods sector — which would allow us to build more refined quantitative models — would be particularly useful.” (Calvo and Vegh, 1999, p.1581)

We add a labor-leisure choice and sector-specific capital accumulation to the model of Section 6. The complete model is laid out in Table 3. The key equation in the model is (B14), the first-order condition for the optimal supply of labor. It is evident from inspection of this condition that a lower rate of crawl raises the return to work by reducing the nominal interest rate and the effective price of consumption. The jump in employment, in turn, increases the marginal product of capital ($F_{KL}, G_{KL} > 0$), spurring firms to invest more. Across steady states, output, employment, and the capital stock in each sector increase by the same percentage amount as the supply of labor. The real exchange rate appreciates in the short and medium run, but returns to its original level in the long run.

Before proceeding further, we call attention to two aspects of the model. First, investment rises only in response to the increase in employment. It follows from this and the separable form of the utility function that the limiting case $s \to \infty$ (perfectly inelastic labor supply) retrieves the model without supply effects. The strength of the supply effects can be measured therefore by comparing the results with corresponding run in Section 7.

The other noteworthy feature of the model is that firms accumulate capital in both
sectors. By contrast, the rest of the ERBS literature restricts investment to the tradables sector. This bothers us. Symmetry is not only more natural, it is also important to the credibility of the results (no pun intended). Keeping capital out of the nontradables sector puts a thumb on the scale, creating a bias toward the desired outcome of strong appreciation of the real exchange rate. The story line is simple. On impact, \( P_n \) jumps upward as higher investment in the tradables sector increases demand for nontraded capital inputs.\(^{22}\) Beyond the short run, expansion in the tradables sector raises real income and either decreases or slows employment growth in the nontradables sector. Higher real income pushes the demand curve in the nontradables sector further to the right, while the transfer of labor to the tradables sector shifts the supply curve to the left. Both factors pull in the direction of more appreciation of the real exchange rate.

Under symmetry, things play out very differently. Since the terms of trade move in favor of the nontradables sector, the supply boom is concentrated there. Thus the pure supply effects serve to moderate appreciation of the real exchange rate. In fact, at the end of the day, they prevent any change in \( P_n \). This does not imply, however, that real appreciation is continuously less than in the model without supply effects. Supply-side expansion takes time to develop, whereas the impact of higher wealth on durables spending is immediate and large. For a while, demand grows more rapidly (ex ante) than supply in the nontradables sector.

6.2 Temporary ERBS

To calibrate the model we set \( s \) so that labor supply increases 5.5-6.5% when ERBS is permanent and \( f \) so that the elasticity of investment with respect to Tobin’s \( q \) equals .25, 1 or 2. (All other parameters take the same values as in Section 6.) The response of labor supply is in line with response assumed in Rebelo and Vegh (1995) and with the responses reported in Roldos (1995) for Mexico’s 1987 Solidarity Pact and Argentina’s 1991 Convertibility Plan.\(^{23}\) The alternative values for the \( q \)-elasticity correspond to low, middle and high-end estimates in the literature on empirical investment functions.\(^{24}\)

Figure 11 and Panel B of Table 2 contain the latest results. The fit with the stylized facts is excellent, including the temporal and sectoral distribution of supply effects. Investment decreases slightly in both sectors, while employment and output rise sharply in the nontrad-
ables sector and fall in the export sector. Real GDP is 2.5-3% higher at the peak of the boom, and recessionary pressures appear 6-9 months before ERBS collapses. On the demand side, although the supply response is temporary and the associated wealth effect small, the consumption boom is significantly stronger than before. The peak increases in durables spending and aggregate consumption are 79-142% and 23-35%; without supply effects, the ranges are 57-97% and 17-22%. Finally, despite rapid supply expansion in the nontradables sector, ERBS is still compatible with pronounced appreciation of the real exchange rate; because the consumption boom is so strong, $P_n$ increases 17-25%.

A. Remarks

Below we comment on (i) the interdependence of the consumption and investment dynamics and (ii) how the results for investment compare to the empirical evidence.

- Since the increase in labor supply raises the marginal product of capital, it might seem odd that investment falls. The decrease in the nominal interest rate, however, lowers the effective price of consumption relative to the price of capital. This creates an incentive to substitute away from fixed investment toward current consumption during the ERBS phase. Substitution between the two assets is easier and the consumption boom stronger when the q-elasticity of investment equals two (see Table 2). Total spending increases less, however, making it more difficult to explain strong appreciation of the real exchange rate.

- At the trough of the cycle, investment is lower by 15-20% in the runs where the q-elasticity equals two and by 2-4% in the runs where the q-elasticity equals .25. Neither result contradicts the stylized facts. The raw numbers for fixed investment are all over the map in ERBS programs (Kiguel and Liviatan, 1992; Reinhart and Vegh, 1995b; Calvo and Vegh, 1999), and the estimated impact is weak and statistically insignificant in Hamaan’s (2001) large panel dataset.

- The stabilization time profile for fixed investment in Calvo and Vegh (1999) shows an initial decline of 7-10% followed by recovery to a temporarily higher level in the post-ERBS period. Our model generates the same investment cycle, with similar numbers, when the q-elasticity of investment spending equals unity (see Figure 11).

- The model suggests two explanations for the mix of positive and negative outcomes in the data. First, as will be seen in Section 6.3, investment increases sharply when ERBS is permanent (or perceived as credible). Thus variations in the response of investment might reflect variations in the credibility of different ERBS programs. Second, it can be argued that part of investment spending should enter the scale variable in the liquidity cost function. This would introduce another pro-investment effect (alongside higher labor supply) that competes against the substitution-toward-durables effect. We conjecture that the sign and the magnitude of the impact on investment in non-credible programs

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would then be sensitive to small variations in the coefficient on $I$ in the liquidity cost function.

B. An Important Refinement

Twenty-five percent appreciation of the real exchange rate is very good by the standards of the literature, but not good enough. While there is always a problem in controlling for other effects, it appears that in some episodes ERBS caused the real exchange rate to appreciate 40-50\% (Calvo and Vegh, 1994b, Table 1).

A refinement of the model helps a great deal here. Suppose firms incur adjustment costs when changing the workforce. This converts $L_n$ and $L_x$ into state variables, producing a smoother, more realistic supply response (there is no jump in nontradables output at $t = 0$).

In Panel C of Table 2 we assume adjustment costs are the same for labor and capital. Overall, the results now match up better with the data. The consumption boom is no less powerful than in the model without supply effects, but appreciation of the real exchange rate rises to 26-38\%.

6.3 Permanent ERBS

"It should be pointed out, however, that for the purposes of our model the wealth effect need not be necessarily large since there is a ‘multiplicative’ effect brought about by the bunching in the purchase of durable goods. Hence, it is conceivable that a relatively small wealth effect may have a large impact on durable goods purchases." (De Gregorio et al., 1998, pp.126-127)

Endowing ERBS with credibility makes the wealth effect stronger but eliminates the incentive to substitute from future to current consumption. The tradeoff is obviously unfavorable. Without intertemporal substitution, the paths for aggregate consumption, durables spending, and the real exchange rate peak much too early and far too low (Figure 12 and Panels D and E of Table 2). This conclusion is extremely robust. Fishing for better numbers, we experimented with alternative values for every parameter in the model. The results always came back looking like Figure 12. Our search did not turn up a single case in which the peak increases for both aggregate consumption and the real exchange rate exceeded 10\%.

These results cast doubt on the thesis of De Gregorio et al. (1998) that the wealth effects associated with credible ERBS can account for the boom phase of the consumption
cycle via their impact on durables spending. Our model of durables expenditure postulates deliberation costs instead of an S-s rule, but this difference is superficial. We get the bunching pattern predicted by their analytical results: the surge in durables expenditure is packed into the first six months of the transition path. The numerical simulations also incorporate large wealth effects — the increase in real income from supply-side expansion is 5.5-6.5%. Nevertheless, the consumption boom lacks height and length. The thesis in De Gregorio et al. isn’t exactly wrong. After all, a consumption boom of 5% is pretty good. But, to repeat, ERBS is special: the stylized facts demand much bigger numbers.

7. Concluding Remarks

Many supporters of the weak credibility (WC) hypothesis probably sympathize with the theorist who complained that “facts are inconvenient things.” In empirical tests, models reliant on the hypothesis have not come close to explaining the dramatic consumption boom seen in ERBS programs. But maybe the problem is with the models as opposed to the hypothesis. The tests were conducted on models that assume all consumption is nondurable. Since the response of nondurables consumption is limited by the low value of the intertemporal elasticity of substitution, it was almost a foregone conclusion that the models would fare poorly when confronted with the data.

In this paper we have used a mix of theory and numerical methods to investigate the explanatory power of the WC hypothesis in a more general model that allows for both durable and nondurable consumer goods. Inclusion of durables is essential for a fair test. Because refrigerators, TVs, etc. provide a service flow for many years, large-scale purchases of durables do not unbalance the consumption path in the same way as a spike in nondurables expenditure. Consequently, low values of the intertemporal elasticity of substitution do not preclude a durables-driven consumption boom. Our results go further and argue that the WC hypothesis is a very promising hypothesis. In numerical simulations based on conservative assumptions about the expenditure share of durables (20%) and wealth effects (none), aggregate consumption increases 17-22% and the real exchange rate appreciates 24-26% during the low-crawl phase. In every case, the boom is powered by spectacular, eye-catching growth in durables spending; when durables are removed from the model, the
peak increase in consumption is only 4-8%.

The model with a standard time-separable utility function suffers from the shortcoming that the path to the peak of the consumption boom has the wrong shape. Consumption grows too fast at the beginning and the end of ERBS and too slow in the middle. This can be fixed by adding habit formation to the model, but only if habit affects deliberation costs involved in purchasing durable goods. For conventional specifications of habit formation, the trajectory of nondurables consumption is hump-shaped but the improvement in the paths of durables spending and aggregate consumption is comparatively modest. Shifting habit formation from the utility function to the deliberation cost function solves the latter problem by strengthening the incentive to smooth the path of durables spending. In the runs with this specification, the paths of durables expenditure and aggregate consumption are steeply sloped and hump-shaped: spending rises apace for two years, then slows and contracts sharply in the last year.

In their influential survey paper, Rebelo and Vegh (1995) assert that supply-side effects are “an essential component in accounting for the stylized facts of exchange-rate-based stabilizations.” We disagree. A pedestrian flexprice model that appeals to weak credibility does an excellent job of explaining the stylized facts provided durables are part of the consumption basket and habit formation is modeled in the right way. But supply effects are helpful if not essential. Variants of the model that allow for endogenous labor supply and sector-specific capital accumulation add another 7-13 percentage points to the consumption boom or another 4-12 percentage points to appreciation of the real exchange rate. Fully armed, the WC hypothesis can account for ERBS episodes where aggregate consumption rose 35% (e.g., Argentina, 1991-1994) and the real exchange rate appreciated 40%.

Supply and wealth effects operate at maximum strength when ERBS is perceived to be credible. Even in a model with durables, however, they are not potent enough to explain the quantitative aspects of the ERBS syndrome. In our model, increases in labor supply and the capital stock buy, at most, a 5% consumption boom and 3-8% appreciation of the real exchange rate. Something else besides standard supply effects must be fueling a large part of the consumption boom in successful ERBS program. Finding this last piece of the puzzle is the main priority for future research.
NOTES

1. Tradable consumption drops below its pre-stabilization level in a single jump at the time
of the policy reversal. Nontradable consumption may fall below its pre-stabilization level
before ERBS is abandoned.

2. The peak increase in consumption occurs at $t = 0$. The solution stated in the text is
obtained by solving the Calvo-Vegh model for small changes. It is approximately correct
for large changes.

3. The Reinhart-Vegh model delivers much larger increases in consumption when the nom-
inal interest rate falls several hundred percentage points (according to their calculations,
1,270 points in the case of Argentina’s Austral Plan). But then doubts arise about how
the change in the nominal interest rate is computed and about the story told by the
cash-in-advance (liquidity costs) constraint: Are Reinhart and Vegh correct in assuming
that the difference between the initial interest rate and the lowest rate observed during
ERBS is close to the average change in the rate? And is it believable that temporary
huge variations in the nominal interest rate generate equally huge variations in the price
of current vs. future consumption?

4. See the discussion of the results for permanent ERBS in Section 6.3.

5. The data for the U.S. and other developed countries indicate that durables spending is
far smoother than predicted by a frictionless Permanent Income/Life Cycle model.

6. This description of fiscal policy is included only to motivate the failure of ERBS. None
of our results depend on the assumption that fiscal policy is completely passive. The
solutions for the paths of consumption and net foreign debt are independent of the path postulated for $g$. (The only restriction on $g$ is that eventually it must adjust to align the
fiscal deficit with seigniorage.)

7. Although $\pi$ returns to its original level, $k$ and $m$ do not. The central bank suffers a
loss in foreign exchange reserves and the increase in net foreign debt reduces aggregate
consumption and holdings of real money balances.

8. Write the term multiplying $(D - \bar{D})$ as

$$\frac{U_{CD}^2}{U_{CC}} - U_{DD} = \frac{U_D}{D} \left[ \left( \frac{U_{CD}D}{U_C} \right) \left( \frac{U_{DC}C}{U_D} \right) \frac{U_C}{U_{CC}C} - \frac{U_{DD}D}{U_D} \right]$$

$$= -\frac{U_D}{D} \left[ \frac{\tau \theta_c + \sigma \theta_d}{\sigma \tau} - \frac{(\tau - \sigma)^2 \theta_d \theta_c}{(\tau \theta_d + \sigma \theta_c) \sigma \tau} \right]$$

$$= -\frac{U_D}{(\tau \theta_d + \sigma \theta_c) D} \left[ \frac{(\tau \theta_d + \sigma \theta_c)(\tau \theta_c + \sigma \theta_d)}{\sigma \tau} - \frac{(\tau - \sigma)^2 \theta_d \theta_c}{\sigma \tau} \right].$$

Mercifully, the term in square brackets simplifies to unity. Moreover, $U_D = U_C (\rho + \delta)$ at
9. The expression for \( f \) is derived from the elasticity formulas for \( \frac{U_{CD}}{U_C} \) and \( -\frac{U_C}{U_{CC}} \).

10. \( S \) and \( C \) decrease proportionately in the long run. This and overshooting imply that the contraction in durables spending is greater than the contraction in nondurables expenditure throughout the post-ERBS adjustment process \( [(S(t) - S_o)/S_o < (C_2 - C_o)/C_o, \ t > T] \).

11. We ignore Arrau et al.'s estimate for Brazil (3.26 is implausibly high) and use the average of Arrau et al. and Reinhart and Veghs' estimates for Argentina and Chile (.27 and .30, respectively). The simple average of the estimated interest elasticities for Chile, Argentina, Mexico, Brazil and Uruguay then works out to .51.

12. The numbers cited here are for failed, noncredible ERBS programs. The 95% confidence interval in the full sample is 14.1-44%.

13. In the Brazilian episode, the path has an early flat stretch much as in Figures 4-7. Consumption rose sharply in year 1, decreased slightly in year 2, and accelerated rapidly in years 3 and 4. See DeGregorio, Guidotti and Vegh (1998, Table 1) and Calvo and Vegh (1994b) for data on the episodes in Mexico, Argentina and Brazil. For data on consumption growth in the Paraguayan and Peruvian programs, see Economic Survey of Latin America and the Caribbean.

14. Following a permanent increase in \( N \) from \( N_o \) to \( N_1 \), the solution to (45) reads \( [H(t) - H_1]/(H_1 - H_o) = e^{-vt} \) (where \( H_1 = N_1 \)). For \( v = 6.5 \) and \( t = .25 \), this gives \( [H(t) - H_1]/(H_1 - H_o) = e^{-1.625t} = .197 \).

15. In the model without habit formation, the dynamic system has two jump variables, \( \omega_1 \) and \( \omega_2 \) (or \( C \) and \( S \)), the multipliers associated with the constraints in equations (4) and (5). (Even though \( \dot{\omega}_1 = 0 \), the computer has to solve for the initial jump in \( \omega_1 \).) When habit formation enters both the utility function and the deliberation cost function, the multipliers associated with the equations governing habit accumulation become part of the dynamic system. This adds two more jump variables to the system.

16. A broad definition of durables includes semi-durables like clothing and footwear. The inclusion of semi-durables and the likelihood that the poorest half of the population pays for most of its durables purchases with cash (even those who are better off may have to make a downpayment to get acceptable credit terms) argues that \( \xi \) is closer to unity than zero. It should also be noted that money may facilitate durables purchases through other changels similar in spirit to the liquidity cost function. De Gregorio et al. (1998) emphasize that the purchase of durable goods is time intensive; if money reduces time spent in transactions related to nondurables consumption, it increases the supply of true leisure time and lowers the total time + money cost of buying a durable good.

17. Recall that we chose the low value .10 in the one-sector model to offset any bias toward
a strong consumption boom caused by the absence of a nontradables sector.

18. Prior to the adoption of ERBS programs in Chile (1978), Uruguay (1978), Argentina (1978), Ecuador (1993), Brazil (1994), and Venezuela (1990, 1997), the ratio of M1 to private consumption ranged from 12% to 17%. At the start of ERBS episodes in Sierre Leone (1988), Zambia (1990), Egypt (1991), Zimbabwe (1993), Nigeria (1990, 1995), and Mozambique (1996), the range was 13% to 23%.

19. There are not many successful ERBS programs. Informed observers contend, however, that many programs enjoyed a high degree of credibility until shortly before they collapsed. See, for example, Dornbusch and Werner (1994), Nazmi (1997), Blejer and Castillo (1998), and Cinquetti (2000).

20. Lahiri (2001) works out analytical (not quantitative) results in a model with a similar specification for the supply-side of the economy. Our model differs in allowing for durable consumer goods, nontraded capital inputs, habit formation, and capital accumulation in the nontradables sector.

21. Across steady states, \( P_n G_K(K_n/L_n) = (\rho + c)P_K(P_n) \), \( F_K(K_x/L_x) = (\rho + c)P_K(P_n) \), and \( P_n G_L(K_n/L_n) = F_L(K_x/L_x) \). These three equations can be solved for \( K_n/L_n \), \( K_x/L_x \), and \( P_n \). None of the variables change in the long run. Under constant returns to scale, they depend only on technology and the rate of time preference.

22. Typically investment is included in the cash-in-advance constraint or the liquidity cost function. This virtually guarantees that investment will increase after the rate of crawl declines.

23. \( s \) equals three. The increase in labor supply is 5.5% when \( \tau = .25 \) and 6.5% when \( \tau = .50 \). Rebelo and Vegh also set \( s \) equal to three but employ a utility function where consumption and leisure are Edgeworth substitutes. Roldos (1995) reports that labor force participation increased 5.2% in Argentina from 1990-1993 and 6.8% in Mexico from 1988-1993.

24. The estimates for developed countries range from .2 to 2.3, with the majority being less than unity (Engel and Foley, 1975; Malkiel et al., 1986; Summers, 1981; Hayashi, 1982; Abel and Blanchard, 1986; Galeoti and Schiantarelli, 1991; Alonso-Borrego and Bertolila, 1994). There are also a few estimates for LDCs, but it is doubtful that the small, thin stock markets in the studies — the source for data on the demand price of capital — accurately reflect information about the fundamentals that drive private investment decisions. A value of two is consistent with Shafik’s (1990) estimate of the elasticity of investment with respect to the supply price of capital in Egypt, and with recent theoretical and empirical work which suggests that previous studies may have substantially underestimated the q-elasticity in developed countries (Barnett and Sakellaris, 1998).

25. Although the empirical evidence is highly mixed, there are well documented cases where ERBS was accompanied by a surge in private investment (e.g., Israel 1985, Mexico 1988, Argentina 1999, and Turkey 2000).
References


Lluch, C. et al., 1977, Patterns in Household Demand and Saving (London, Oxford University Press).


<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
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<td>Intertemporal elasticity of substitution</td>
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<td>Share of nontradables in durables consumption</td>
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<td>Share of durables in total consumption spending</td>
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<td>Habit persistence parameter</td>
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<td>Slope of the loan supply schedule (κ)</td>
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Table 2: Peak increases in durables spending, aggregate consumption, and the real exchange rate in different variants of the model with a nontradables sector.\footnote{1}

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<thead>
<tr>
<th>Panel A: No Supply Effects</th>
<th>Variant</th>
<th>Durables Spending</th>
<th>Total Consumption</th>
<th>Real Exchange Rate</th>
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Table 2 (continued)

**Panel D: Permanent ERBS with Supply Effects**

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<td>$\Psi = 10$</td>
<td>Base Run</td>
<td>12.0</td>
<td>5.1</td>
</tr>
<tr>
<td></td>
<td>$\sigma = 0.25$</td>
<td>14.8</td>
<td>5.3</td>
</tr>
</tbody>
</table>

**Panel E: Permanent ERBS with Supply Effects and Adjustment Costs to Changing Employment**

<table>
<thead>
<tr>
<th>Variant</th>
<th>Durables Spending</th>
<th>Total Consumption</th>
<th>Real Exchange Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Psi = 1$</td>
<td>Base Run</td>
<td>6.8</td>
<td>3.6</td>
</tr>
<tr>
<td></td>
<td>$\sigma = 0.25$</td>
<td>7.7</td>
<td>3.6</td>
</tr>
<tr>
<td></td>
<td>$\tau = 0.50$</td>
<td>7.2</td>
<td>3.7</td>
</tr>
<tr>
<td>$\Psi = 2, \sigma = 0.25$</td>
<td>8.7</td>
<td>3.9</td>
<td>8.0</td>
</tr>
<tr>
<td>$\Psi = 10, \sigma = 0.25$</td>
<td>11.4</td>
<td>4.6</td>
<td>7.0</td>
</tr>
</tbody>
</table>

1 Percentage increases relative to pre-ERBS steady state.

2 In all of the panels, $\tau = 0.25$ and $\sigma = 0.75$ in the Base Run.

3 $\Psi$ is the elasticity of investment with respect to Tobin’s q.

4 Figures are for the peak increase in total consumption in the first five years. In the very long run, total consumption increases by the same amount as real output (5.5% when $\tau = 0.25$ and 6.5% when $\tau = 0.50$).
Table 3: The Model with Supply Effects

\[ Q_x = F(K_x, L_x) = [a_3 L_x^{(\sigma-1)/\sigma} + a_4 K_x^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)} \]  
\[ Q_n = G(K_n, L_n) = [a_5 L_n^{(\sigma-1)/\sigma} + a_6 K_n^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)} \]  

\[ F_L = w \]  
\[ P_n G_L = w \]  
\[ P_d = 1 + a_7 P_n \]

Private Agent’s Optimization Problem

\[ \max \int_0^\infty \left[ \frac{(N/H_1^\alpha)^{1/\tau} - a_8 L^s - x (S/H_2 - 1)^2 H_2}{1 - 1/\tau} \right] e^{-\rho t} dt, \quad s > 1, \]  

subject to

\[ \dot{A} = P_n G(K_n, L_n) + F(K_x, L - L_n) + \bar{g} + r(A - m) - (E + P_d S) \left[ 1 + \frac{m}{E + P_d S} \right] \]  
\[ -P_k \left[ I_n + f \left( I_n / K_n - c \right)^2 K_n \right] - P_k \left[ I_x + f \left( I_x / K_x - c \right)^2 K_x \right] - \chi m, \]
\[ \dot{D} = S - \delta D, \]
\[ \dot{H}_1 = v(B - H_1), \]
\[ \dot{H}_2 = v(S - H_2), \]
\[ \dot{K}_n = I_n - cK_n, \]
\[ \dot{K}_x = I_x - cK_x, \]

where

\[ N \equiv \{a_1 [E/c(P_n)]^{\sigma-1/\sigma} + a_2 D^{(\sigma-1)/\sigma}\}^{\sigma/(\sigma-1)}. \]

First-order conditions and co-state equations:

\[ \left( \frac{N}{H_1^\alpha} \right)^{-1/\tau} \frac{N_E}{H_1^\alpha} + \phi_3 v N_E = \phi_4 (1 + L - L'm/X), \]  
\[ a_8 s L_x^{s-1} = \phi_1 F_L, \]  

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\[ P_n G_L = F_L, \]  
\[ -L' = r + \chi, \]  
\[ \phi_2 + \phi_4 v = \phi_1 P_d (1 + L - L'm/X) + x(S/H_2 - \delta), \]  
\[ \phi_5 = \phi_1 P_k [1 + f(I_n/K_n - c)], \]  
\[ \phi_6 = \phi_1 P_k [1 + f(I_x/K_x - c)], \]  
\[ \dot{\phi}_1 = \phi_1 (\rho - r), \]  
\[ \dot{\phi}_2 = (\rho + \delta) \phi_2 - \phi_3 \nu N_D - \left( \frac{N}{H_1^\alpha} \right)^{-1/\tau} \frac{N_D}{H_1^\alpha}, \]  
\[ \dot{\phi}_3 = (\rho + v) \phi_3 + \left( \frac{N}{H_1^\alpha} \right)^{-1/\tau} \left( \frac{N}{H_1^\alpha} \frac{\alpha N}{H_1^\alpha + 1} \right), \]  
\[ \dot{\phi}_4 = (\rho + v) \phi_4 + x \left( \frac{S/H_2 - 1}{2} \right)^2 - x(S/H_2 - 1) \frac{S}{H_2}, \]  
\[ \dot{\phi}_5 = (\rho + c) \phi_5 - \phi_1 \left[ F_K + P_k f(I_x/K_x - c) \left( \frac{I_x}{K_x} - \frac{I_x/K_x - c}{2} \right) \right], \]  
\[ \dot{\phi}_6 = (\rho + c) \phi_6 - \phi_1 \left[ P_n G_K + P_k f(I_n/K_n - c) \left( \frac{I_n}{K_n} - \frac{I_n/K_n - c}{2} \right) \right]. \]  

where \( \phi_1- \phi_6 \) are the multipliers attached to the constraints in (B7)-(B12).

\[ \dot{Z} = P_n Q_n + Q_x + rZ - E - P_d S - P_k \left[ I_n + I_x + f \frac{(I_n/K_n - c)^2 K_n}{2} + f \frac{(I_x/K_x - c)^2 K_x}{2} \right] \]

\[ C_n + a_7 S = Q_n \]  
\[ C_n = E \frac{k_1 P_n^{-\beta}}{k_1^\beta + k_1^\beta P_n^{1-\beta}} \]  
\[ r = \rho + \kappa \left( \frac{b}{P_n Q_n + Q_x} - \frac{b_o}{P_{n,o} Q_{n,o} + Q_{x,o}} \right), \quad \kappa < 0 \]
Figure 1: The transition path in the benchmark case of a separable utility function.
Figure 2: The transition path when durables and nondurables are Edgeworth substitutes.
Figure 3: The transition path when durables and nondurables are Edgeworth complements.
Panel A: $\Omega = 5$.

Panel B: $\Omega = 10$.

Figure 4: Transition path when $\sigma = \tau = 0.25$. 
Panel A: $\Omega = 5$.

Panel B: $\Omega = 10$.

Figure 5: Transition path when $\sigma = \tau = .50$. 
Panel A: $\Omega = 5$.

Panel B: $\Omega = 10$.

Figure 6: Transition path when durables and nondurables are Edgeworth Complements ($\tau = .50$ and $\sigma = .25$).
Panel A: $\Omega = 5$.

Panel B: $\Omega = 10$.

Figure 7: Transition path when durables and nondurables are Edgeworth Substitutes ($\sigma = .75$, $\tau = .25$).
Panel A: $v = .5$.

Panel B: $v = 6.5$.

Figure 8: Transition path with the multiplicative specification of habit formation (--- is the path without habit formation).
Panel A: All durables spending enters the liquidity cost function (---is the path without habit formation).

Panel B: Transition path when $\xi = .50$.

Panel C: Transition path when $\xi = .25$.

Figure 9: Transition Path when habit formation affects deliberation costs.
Figure 10: Base Run for the model with a nontradables sector.
Figure 11: Base Run for Temporary ERBS with supply effects (q-elasticity of investment equals unity).
Figure 12: Base Run for Permanent ERBS with supply effects (q-elasticity of investment equals unity).